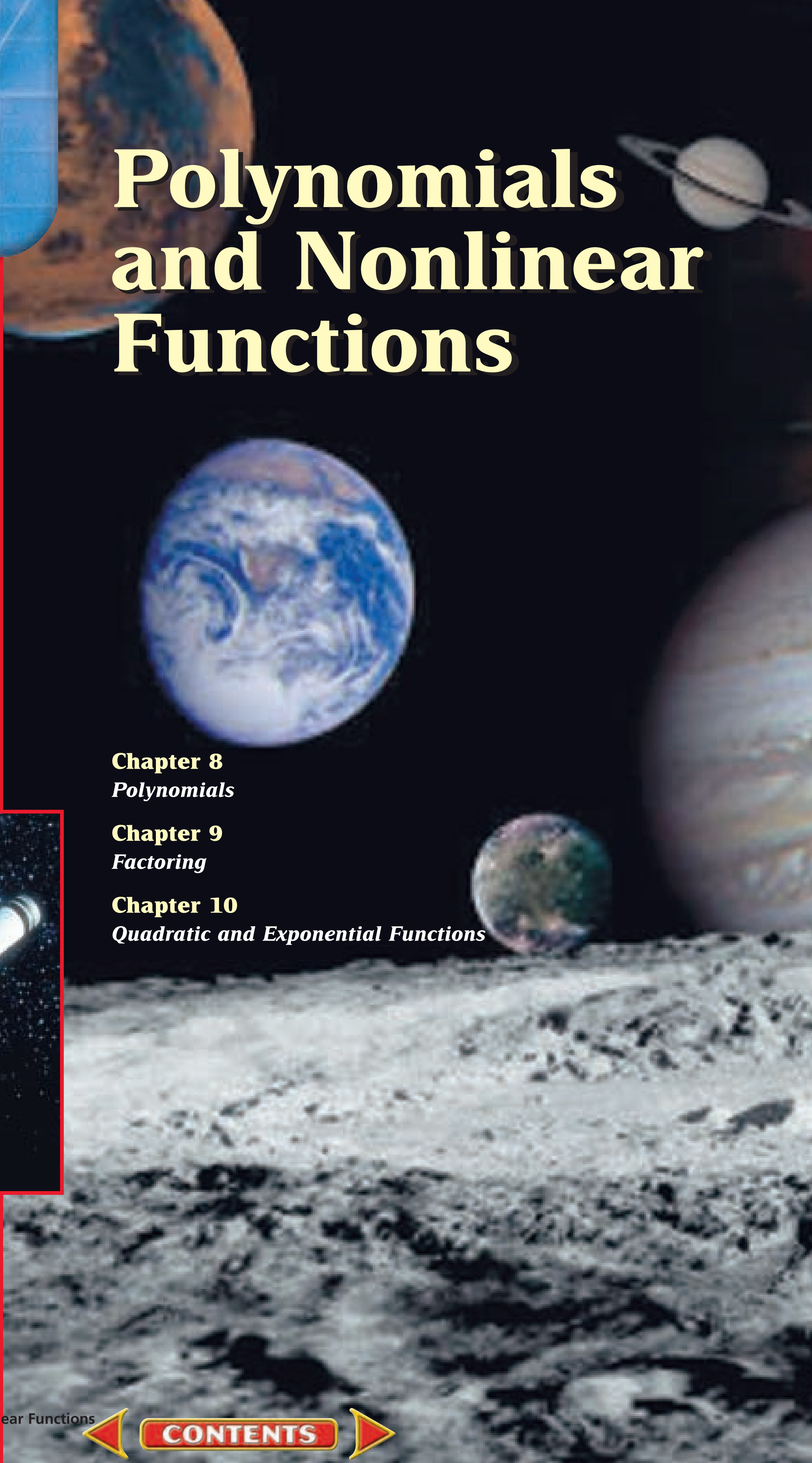
UNIT

3

Polynomials and Nonlinear Functions



Not all real-world situations can be modeled using a linear function. In this unit, you will learn about polynomials and nonlinear functions.

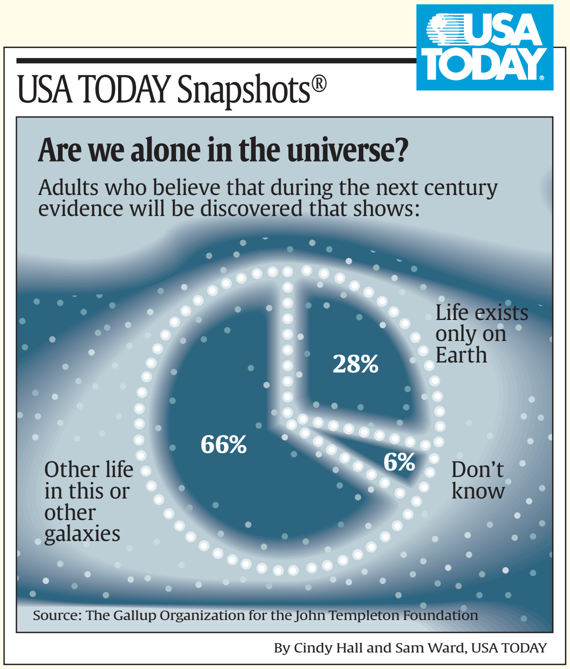
CONTENTS

WebQuest Internet Project

Pluto Is Falling From Status as Distant Planet

Source: *USA TODAY,* March 28, 2001

"Like any former third-grader, Catherine Beyhl knows that the solar system has nine planets, and she knows a phrase to help remember their order: 'My Very Educated Mother Just Served Us Nine Pizzas.' But she recently visited the American Museum of Natural History's glittering new astronomy hall at the Hayden Planetarium and found only eight scale models of the planets. No Pizza—no Pluto." In this project, you will examine how scientific notation, factors, and graphs are useful in presenting information about the planets.



Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 3.

Lesson	8-3	9-1	10-2
Page	429	479	537

Chapter

8 Polynomials

What You'll Learn

- **Lessons 8-1 and 8-2** Find products and quotients of monomials.
- **Lesson 8-3** Express numbers in scientific and standard notation.
- **Lesson 8-4** Find the degree of a polynomial and arrange the terms in order.
- **Lessons 8-5 through 8-7** Add, subtract, and multiply polynomial expressions.
- **Lesson 8-8** Find special products of binomials.

Key Vocabulary

- **monomial** (p. 410)
- **scientific notation** (p. 425)
- **polynomial** (p. 432)
- **binomial** (p. 432)
- **FOIL method** (p. 453)

Why It's Important

Operations with polynomials, including addition, subtraction, and multiplication, form the foundation for solving equations that involve polynomials. In addition, polynomials are used to model many real-world situations. *In Lesson 8-6, you will learn how to find the distance that runners on a curved track should be staggered.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 8.

For Lessons 8-1 and 8-2 — Exponential Notation

Write each expression using exponents. *(For review, see Lesson 1-1.)*

1. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ **2.** $3 \cdot 3 \cdot 3 \cdot 3$ **3.** $5 \cdot 5$ **4.** $x \cdot x \cdot x$

5. $a \cdot a \cdot a \cdot a \cdot a \cdot a$ **6.** $x \cdot x \cdot y \cdot y \cdot y$ **7.** $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ **8.** $\frac{a}{b} \cdot \frac{a}{b} \cdot \frac{c}{d} \cdot \frac{c}{d} \cdot \frac{c}{d}$

For Lessons 8-1 and 8-2 — Evaluating Powers

Evaluate each expression. *(For review, see Lesson 1-1.)*

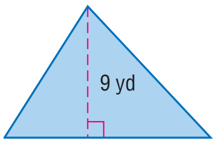
9. 3^2 **10.** 4^3 **11.** 5^2 **12.** 10^4

13. $(-6)^2$ **14.** $(-3)^3$ **15.** $\left(\frac{2}{3}\right)^4$ **16.** $\left(-\frac{7}{8}\right)^2$

For Lessons 8-1, 8-2, and 8-5 through 8-8 — Area and Volume

Find the area or volume of each figure shown below. *(For review, see pages 813–817.)*

17.



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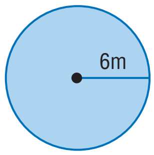

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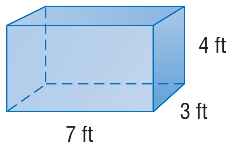

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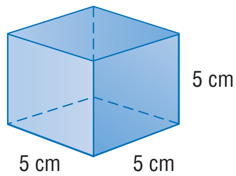
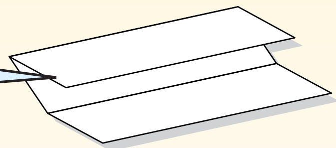


FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about polynomials. Begin with a sheet of 11" by 17" paper.

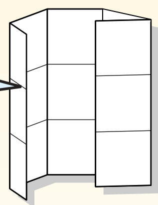
Step 1 Fold

Fold in thirds lengthwise.



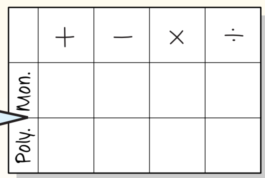
Step 2 Open and Fold

Fold a 2" tab along the width. Then fold the rest in fourths.



Step 3 Label

Draw lines along folds and label as shown.



Reading and Writing As you read and study the chapter, write examples and notes for each operation.

8-1 Multiplying Monomials

What You'll Learn

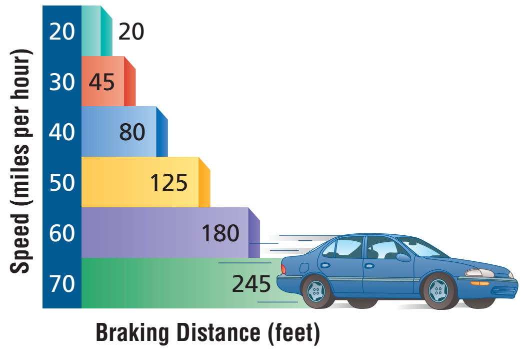
- Multiply monomials.
- Simplify expressions involving powers of monomials.

Vocabulary

- monomial
- constant

Why does doubling speed quadruple braking distance?

The table shows the braking distance for a vehicle at certain speeds. If s represents the speed in miles per hour, then the approximate number of feet that the driver must apply the brakes is $\frac{1}{20}s^2$. Notice that when speed is doubled, the braking distance is quadrupled.



Source: *British Highway Code*

MULTIPLY MONOMIALS An expression like $\frac{1}{20}s^2$ is called a monomial. A **monomial** is a number, a variable, or a product of a number and one or more variables. An expression involving the division of variables is not a monomial. Monomials that are real numbers are called **constants**.

Example 1 Identify Monomials

Determine whether each expression is a monomial. Explain your reasoning.

	Expression	Monomial?	Reason
a.	-5	yes	-5 is a real number and an example of a constant.
b.	$p + q$	no	The expression involves the addition, not the product, of two variables.
c.	x	yes	Single variables are monomials.
d.	$\frac{c}{d}$	no	The expression is the quotient, not the product, of two variables.
e.	$\frac{abc^8}{5}$	yes	$\frac{abc^8}{5} = \frac{1}{5}abc^8$. The expression is the product of a number, $\frac{1}{5}$, and three variables.

Study Tip

Reading Math

The expression x^n is read *x to the nth power*.

Recall that an expression of the form x^n is called a *power* and represents the product you obtain when x is used as a factor n times. The number x is the *base*, and the number n is the *exponent*.

$$2^5 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{5 factors}} \text{ or } 32$$

exponent → 5; base → 2

In the following examples, the definition of a power is used to find the products of powers. Look for a pattern in the exponents.

$$2^3 \cdot 2^5 = \underbrace{\overbrace{2 \cdot 2 \cdot 2}^{3\text{ factors}} \cdot \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{5\text{ factors}}}_{3 + 5\text{ or }8\text{ factors}} \text{ or } 2^8$$

$$3^2 \cdot 3^4 = \underbrace{\overbrace{3 \cdot 3}^{2\text{ factors}} \cdot \overbrace{3 \cdot 3 \cdot 3 \cdot 3}^{4\text{ factors}}}_{2 + 4\text{ or }6\text{ factors}} \text{ or } 3^6$$

These and other similar examples suggest the property for multiplying powers.

Key Concept *Product of Powers*

- **Words** To multiply two powers that have the same base, add the exponents.
- **Symbols** For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.
- **Example** $a^4 \cdot a^{12} = a^{4+12}$ or a^{16}

Example 2 Product of Powers

Simplify each expression.

a. $(5x^7)(x^6)$

$(5x^7)(x^6) = (5)(1)(x^7 \cdot x^6)$ Commutative and Associative Properties

$= (5 \cdot 1)(x^{7+6})$ Product of Powers

$= 5x^{13}$ Simplify.

b. $(4ab^6)(-7a^2b^3)$

$(4ab^6)(-7a^2b^3) = (4)(-7)(a \cdot a^2)(b^6 \cdot b^3)$ Commutative and Associative Properties

$= -28(a^{1+2})(b^{6+3})$ Product of Powers

$= -28a^3b^9$ Simplify.

Study Tip

Power of 1

Recall that a variable with no exponent indicated can be written as a power of 1. For example, $x = x^1$ and $ab = a^1b^1$.

POWERS OF MONOMIALS You can also look for a pattern to discover the property for finding the power of a power.

$$(4^2)^5 = \overbrace{(4^2)(4^2)(4^2)(4^2)(4^2)}^{5\text{ factors}} = 4^{2+2+2+2+2} = 4^{10}$$

$$(z^8)^3 = \overbrace{(z^8)(z^8)(z^8)}^{3\text{ factors}} = z^{8+8+8} = z^{24}$$

Apply rule for Product of Powers.

Therefore, $(4^2)^5 = 4^{10}$ and $(z^8)^3 = z^{24}$. These and other similar examples suggest the property for finding the power of a power.

Key Concept *Power of a Power*

- **Words** To find the power of a power, multiply the exponents.
- **Symbols** For any number a and all integers m and n, $(a^m)^n = a^{m \cdot n}$.
- **Example** $(k^5)^9 = k^{5 \cdot 9}$ or k^{45}

Study Tip

Look Back

To review **using a calculator to find a power of a number**, see Lesson 1-1.

Example 3 Power of a Power

Simplify $((3^2)^3)^2$.

$((3^2)^3)^2 = (3^{2 \cdot 3})^2$ Power of a Power

$= (3^6)^2$ Simplify.

$= 3^{6 \cdot 2}$ Power of a Power

$= 3^{12}$ or 531,441 Simplify.

www.algebra1.com/extra_examples

Look for a pattern in the examples below.

$$(xy)^4 = (xy)(xy)(xy)(xy) = (x \cdot x \cdot x \cdot x)(y \cdot y \cdot y \cdot y) = x^4y^4$$

$$(6ab)^3 = (6ab)(6ab)(6ab) = (6 \cdot 6 \cdot 6)(a \cdot a \cdot a)(b \cdot b \cdot b) = 6^3a^3b^3 \text{ or } 216a^3b^3$$

These and other similar examples suggest the following property for finding the power of a product.

Study Tip

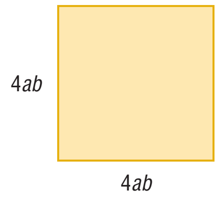
Powers of Monomials
Sometimes the rules for the Power of a Power and the Power of a Product are combined into one rule.
$(a^mb^n)^p = a^{mp}b^{np}$

Key Concept — *Power of a Product*

- **Words** To find the power of a product, find the power of each factor and multiply.
- **Symbols** For all numbers a and b and any integer m, $(ab)^m = a^mb^m$.
- **Example** $(-2xy)^3 = (-2)^3x^3y^3$ or $-8x^3y^3$

Example 4 *Power of a Product*

GEOMETRY **Express the area of the square as a monomial.**



Area	$= s^2$	Formula for the area of a square
	$= (4ab)^2$	$s = 4ab$
	$= 4^2a^2b^2$	Power of a Product
	$= 16a^2b^2$	Simplify.

The area of the square is $16a^2b^2$ square units.

The properties can be used in combination to simplify more complex expressions involving exponents.

Concept Summary — *Simplifying Monomial Expressions*

To *simplify* an expression involving monomials, write an equivalent expression in which:

- each base appears exactly once,
- there are no powers of powers, and
- all fractions are in simplest form.

Example 5 *Simplify Expressions*

Simplify $\left(\frac{1}{3}xy^4\right)^2[(-6y)^2]^3$.

$\left(\frac{1}{3}xy^4\right)^2[(-6y)^2]^3$	$= \left(\frac{1}{3}xy^4\right)^2(-6y)^6$	Power of a Power
	$= \left(\frac{1}{3}\right)^2x^2(y^4)^2(-6)^6y^6$	Power of a Product
	$= \frac{1}{9}x^2y^8(46{,}656)y^6$	Power of a Power
	$= \frac{1}{9}(46{,}656)x^2 \cdot y^8 \cdot y^6$	Commutative Property
	$= 5184x^2y^{14}$	Product of Powers

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of an expression that can be simplified using each property. Then simplify each expression.
   a. Product of Powers **b.** Power of a Power **c.** Power of a Product
2. **Determine** whether each pair of monomials is equivalent. Explain.
   a. $5m^2$ and $(5m)^2$ **b.** $(yz)^4$ and y^4z^4
   c. $-3a^2$ and $(-3a)^2$ **d.** $2(c^7)^3$ and $8c^{21}$
3. **FIND THE ERROR** Nathan and Poloma are simplifying $(5^2)(5^9)$.

Nathan	Poloma
$(5^2)(5^9) = (5 \cdot 5)^{2+9}$	$(5^2)(5^9) = 5^{2+9}$
$= 25^{11}$	$= 5^{11}$

Who is correct? Explain your reasoning.

Guided Practice

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

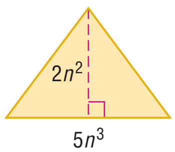
4. $5 - 7d$
5. $\frac{4a}{3b}$
6. n

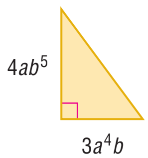
Simplify.

7. $x(x^4)(x^6)$
8. $(4a^4b)(9a^2b^3)$
9. $[(2^3)^2]^3$
10. $(3y^5z)^2$
11. $(-4mn^2)(12m^2n)$
12. $(-2v^3w^4)^3(-3vw^3)^2$

Application

GEOMETRY **Express the area of each triangle as a monomial.**

13. 

14. 

Practice and Apply

Homework Help

For Exercises	See Examples
15–20	1
21–48	2, 3, 5
49–54	4

Extra Practice

See page 837.

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

15. 12
16. $4x^3$
17. $a - 2b$
18. $4n + 5m$
19. $\frac{x}{y^2}$
20. $\frac{1}{5}abc^{14}$

Simplify.

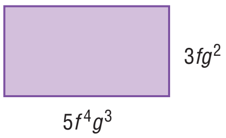
21. $(ab^4)(ab^2)$
22. $(p^5q^4)(p^2q)$
23. $(-7c^3d^4)(4cd^3)$
24. $(-3j^7k^5)(-8jk^8)$
25. $(5a^2b^3c^4)(6a^3b^4c^2)$
26. $(10xy^5z^3)(3x^4y^6z^3)$
27. $(9pq^7)^2$
28. $(7b^3c^6)^3$
29. $[(3^2)^4]^2$
30. $[(4^2)^3]^2$
31. $(0.5x^3)^2$
32. $(0.4h^5)^3$
33. $\left(-\frac{3}{4}c\right)^3$
34. $\left(\frac{4}{5}a^2\right)^2$
35. $(4cd)^2(-3d^2)^3$
36. $(-2x^5)^3(-5xy^6)^2$
37. $(2ag^2)^4(3a^2g^3)^2$
38. $(2m^2n^3)^3(3m^3n)^4$
39. $(8y^3)(-3x^2y^2)\left(\frac{3}{8}xy^4\right)$
40. $\left(\frac{4}{7}m\right)^2(49m)(17p)\left(\frac{1}{34}p^5\right)$

41. Simplify the expression $(-2b^3)^4 - 3(-2b^4)^3$.

42. Simplify the expression $2(-5y^3)^2 + (-3y^3)^3$.

GEOMETRY **Express the area of each figure as a monomial.**

43.



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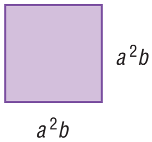

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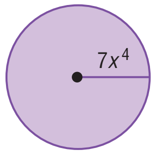
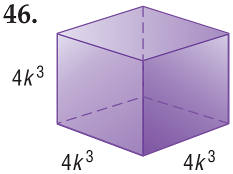


GEOMETRY **Express the volume of each solid as a monomial.**

46.



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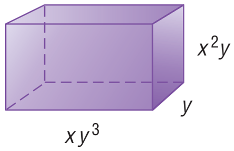

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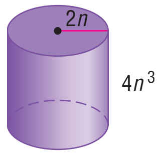


TELEPHONES **For Exercises 49 and 50, use the following information.**
The first transatlantic telephone cable has 51 amplifiers along its length. Each amplifier strengthens the signal on the cable 10^6 times.

49. After it passes through the second amplifier, the signal has been boosted $10^6 \cdot 10^6$ times. Simplify this expression.

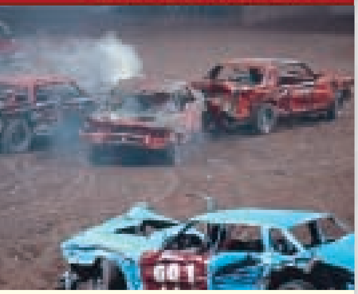
50. Represent the number of times the signal has been boosted after it has passed through the first four amplifiers as a power of 10^6. Then simplify the expression.

DEMOLITION DERBY **For Exercises 51 and 52, use the following information.**
When a car hits an object, the damage is measured by the collision impact. For a certain car, the collision impact I is given by $I = 2s^2$, where s represents the speed in kilometers per minute.

51. What is the collision impact if the speed of the car is 1 kilometer per minute? 2 kilometers per minute? 4 kilometers per minute?

52. As the speed doubles, explain what happens to the collision impact.

More About. . .



Demolition Derby

In a demolition derby, the winner is not the car that finishes first but the last car still moving under its own power.

Source: *Smithsonian Magazine*

TEST TAKING **For Exercises 53 and 54, use the following information.**
A history test covers two chapters. There are 2^{12} ways to answer the 12 true-false questions on the first chapter and 2^{10} ways to answer the 10 true-false questions on the second chapter.

53. How many ways are there to answer all 22 questions on the test? (*Hint*: Find the product of 2^{12} and 2^{10}.)

54. If a student guesses on each question, what is the probability of answering all questions correctly?

CRITICAL THINKING **Determine whether each statement is *true* or *false*. If true, explain your reasoning. If false, give a counterexample.**

55. For any real number a, $(-a)^2 = -a^2$.

56. For all real numbers a and b, and all integers m, n, and p, $(a^m b^n)^p = a^{mp} b^{np}$.

57. For all real numbers a, b, and all integers n, $(a + b)^n = a^n + b^n$.

58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why does doubling speed quadruple braking distance?

Include the following in your answer:

- the ratio of the braking distance required for a speed of 40 miles per hour and the braking distance required for a speed of 80 miles per hour, and
- a comparison of the expressions $\frac{1}{20}s^2$ and $\frac{1}{20}(2s)^2$.

Standardized Test Practice

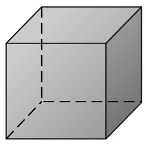
59. $4^2 \cdot 4^5 = ?$

Ⓐ 16^7 Ⓑ 8^7 Ⓒ 4^{10} Ⓓ 4^7

60. Which of the following expressions represents the volume of the cube?

Ⓐ $15x^3$ Ⓑ $25x^2$

Ⓒ $25x^3$ Ⓓ $125x^3$



$5x$

Maintain Your Skills

Mixed Review

Solve each system of inequalities by graphing. *(Lesson 7-5)*

61. $y \leq 2x + 2$
$y \geq -x - 1$

62. $y \geq x - 2$
$y < 2x - 1$

63. $x > -2$
$y < x + 3$

Use elimination to solve each system of equations. *(Lesson 7-4)*

64. $-4x + 5y = 2$
$x + 2y = 6$

65. $3x + 4y = -25$
$2x - 3y = 6$

66. $x + y = 20$
$0.4x + 0.15y = 4$

Solve each compound inequality. Then graph the solution set. *(Lesson 6-4)*

67. $4 + h \leq -3$ or $4 + h \geq 5$

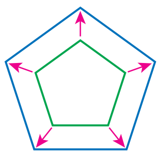
68. $4 < 4a + 12 < 24$

69. $14 < 3h + 2 < 2$

70. $2m - 3 > 7$ or $2m + 7 > 9$

Determine whether each transformation is a *reflection*, *translation*, *dilation*, or *rotation*. *(Lesson 4-2)*

71.



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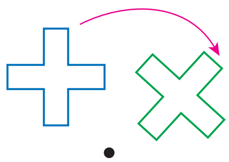

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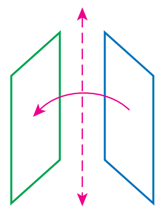


74. **TRANSPORTATION** Two trains leave York at the same time, one traveling north, the other south. The northbound train travels at 40 miles per hour and the southbound at 30 miles per hour. In how many hours will the trains be 245 miles apart? *(Lesson 3-7)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review **simplifying fractions**, see pages 798 and 799.)*

75. $\frac{2}{6}$

76. $\frac{3}{15}$

77. $\frac{10}{5}$

78. $\frac{27}{9}$

79. $\frac{14}{36}$

80. $\frac{9}{48}$

81. $\frac{44}{32}$

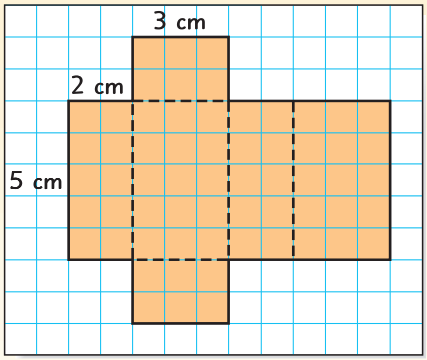
82. $\frac{45}{18}$

Algebra Activity

A Follow-Up of Lesson 8-1

Investigating Surface Area and Volume

Collect the Data



- Cut out the pattern shown from a sheet of centimeter grid paper. Fold along the dashed lines and tape the edges together to form a rectangular prism with dimensions 2 centimeters by 5 centimeters by 3 centimeters.
- Find the surface area *SA* of the prism by counting the squares on all the faces of the prism or by using the formula $SA = 2w\ell + 2wh + 2\ell h$, where w is the width, ℓ is the length, and h is the height of the prism.
- Find the volume V of the prism by using the formula $V = \ell wh$.
- Now construct another prism with dimensions that are 2 times each of the dimensions of the first prism, or 4 centimeters by 10 centimeters by 6 centimeters.
- Finally, construct a third prism with dimensions that are 3 times each of the dimensions of the first prism, or 6 centimeters by 15 centimeters by 9 centimeters.

Analyze the Data

1. Copy and complete the table using the prisms you made.

Prism	Dimensions	Surface Area (cm^2)	Volume (cm^3)	Surface Area Ratio $\left(\frac{SA \text{ of New}}{SA \text{ of Original}}\right)$	Volume Ratio $\left(\frac{V \text{ of New}}{V \text{ of Original}}\right)$
Original	2 by 5 by 3	62	30	——	——
A	4 by 10 by 6				
B	6 by 15 by 9				

2. Make a prism with different dimensions from any in this activity. Repeat the steps in **Collect the Data**, and make a table similar to the one in Exercise 1.

Make a Conjecture

3. Suppose you multiply each dimension of a prism by 2. What is the ratio of the surface area of the new prism to the surface area of the original prism? What is the ratio of the volumes?

4. If you multiply each dimension of a prism by 3, what is the ratio of the surface area of the new prism to the surface area of the original? What is the ratio of the volumes?

5. Suppose you multiply each dimension of a prism by a. Make a conjecture about the ratios of surface areas and volumes.

Extend the Activity

6. Repeat the steps in **Collect the Data** and **Analyze the Data** using cylinders. To start, make a cylinder with radius 4 centimeters and height 5 centimeters. To compute surface area *SA* and volume V, use the formulas $SA = 2\pi r^2 + 2\pi rh$ and $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. Do the conjectures you made in Exercise 5 hold true for cylinders? Explain.

8-2 Dividing Monomials

What You'll Learn

- Simplify expressions involving the quotient of monomials.
- Simplify expressions containing negative exponents.

Vocabulary

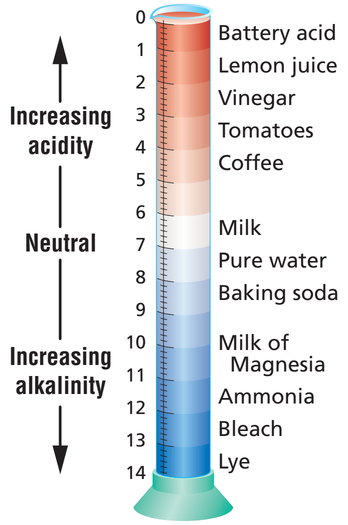
- zero exponent
- negative exponent

How can you compare pH levels?

To test whether a solution is a *base* or an *acid*, chemists use a pH test. This test measures the concentration c of hydrogen ions (in moles per liter) in the solution.

$$c = \left(\frac{1}{10}\right)^{\text{pH}}$$

The table gives examples of solutions with various pH levels. You can find the quotient of powers and use negative exponents to compare measures on the pH scale.



Source: U.S. Geological Survey

QUOTIENTS OF MONOMIALS In the following examples, the definition of a power is used to find quotients of powers. Look for a pattern in the exponents.

$$\frac{4^5}{4^3} = \frac{\overbrace{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}\cdot 4\cdot 4}^{5\text{ factors}}}{\underbrace{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}}_{3\text{ factors}}} = \underbrace{4\cdot 4}_{5-3\text{ or }2\text{ factors}} \text{ or } 4^2$$

$$\frac{3^6}{3^2} = \frac{\overbrace{\cancel{3}\cdot\cancel{3}\cdot 3\cdot 3\cdot 3\cdot 3}^{6\text{ factors}}}{\underbrace{\cancel{3}\cdot\cancel{3}}_{2\text{ factors}}} = \underbrace{3\cdot 3\cdot 3\cdot 3}_{6-2\text{ or }4\text{ factors}} \text{ or } 3^4$$

These and other similar examples suggest the following property for dividing powers.

Key Concept — Quotient of Powers

- **Words** To divide two powers that have the same base, subtract the exponents.
- **Symbols** For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
- **Example** $\frac{b^{15}}{b^7} = b^{15-7}$ or b^8

Example 1 Quotient of Powers

Simplify $\frac{a^5b^8}{ab^3}$. Assume that a and b are not equal to zero.

$\frac{a^5b^8}{ab^3} = \left(\frac{a^5}{a}\right)\left(\frac{b^8}{b^3}\right)$ Group powers that have the same base.

$= (a^{5-1})(b^{8-3})$ Quotient of Powers

$= a^4b^5$ Simplify.

In the following example, the definition of a power is used to compute the power of a quotient. Look for a pattern in the exponents.

$$\left(\frac{2}{5}\right)^3 = \underbrace{\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)}_{\text{3 factors}} = \frac{\overbrace{2 \cdot 2 \cdot 2}^{\text{3 factors}}}{\underbrace{5 \cdot 5 \cdot 5}_{\text{3 factors}}} \text{ or } \frac{2^3}{5^3}$$

This and other similar examples suggest the following property.

Key Concept — *Power of a Quotient*

- **Words** To find the power of a quotient, find the power of the numerator and the power of the denominator.
- **Symbols** For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.
- **Example** $\left(\frac{c}{d}\right)^5 = \frac{c^5}{d^5}$

Example 2 *Power of a Quotient*

Simplify $\left(\frac{2p^2}{3}\right)^4$.

$\left(\frac{2p^2}{3}\right)^4 = \frac{(2p^2)^4}{3^4}$ Power of a Quotient

$= \frac{2^4(p^2)^4}{3^4}$ Power of a Product

$= \frac{16p^8}{81}$ Power of a Power

NEGATIVE EXPONENTS A graphing calculator can be used to investigate expressions with 0 as an exponent as well as expressions with negative exponents.

Graphing Calculator Investigation

Zero Exponent and Negative Exponents

Use the [^] key on a TI-83 Plus to evaluate expressions with exponents.

Think and Discuss

1. Copy and complete the table below.

Power	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Value									

2. Describe the relationship between each pair of values.
   a. 2^4 and 2^{-4} **b.** 2^3 and 2^{-3} **c.** 2^2 and 2^{-2} **d.** 2^1 and 2^{-1}
3. **Make a Conjecture** as to the fractional value of 5^{-1}. Verify your conjecture using a calculator.
4. What is the value of 5^0?
5. What happens when you evaluate 0^0?

Study Tip

Graphing Calculator

To express a value as a fraction, press

[MATH] [ENTER] [ENTER].

To understand why a calculator gives a value of 1 for 2^0, study the two methods used to simplify $\frac{2^4}{2^4}$.

Method 1

$\frac{2^4}{2^4} = 2^{4-4}$ Quotient of Powers

$= 2^0$ Subtract.

Method 2

$\frac{2^4}{2^4} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}}}$ Definition of powers

$= 1$ Simplify.

Since $\frac{2^4}{2^4}$ cannot have two different values, we can conclude that $2^0 = 1$.

Study Tip

Alternative Method

Another way to look at the problem of simplifying $\frac{2^4}{2^4}$ is to recall that any nonzero number divided by itself is 1: $\frac{2^4}{2^4} = \frac{16}{16}$ or 1.

Key Concept — Zero Exponent

- **Words** Any nonzero number raised to the zero power is 1.
- **Symbols** For any nonzero number a, $a^0 = 1$.
- **Example** $(-0.25)^0 = 1$

Example 3 Zero Exponent

Simplify each expression. Assume that x and y are not equal to zero.

a. $\left(-\frac{3x^5y}{8xy^7}\right)^0$

$\left(-\frac{3x^5y}{8xy^7}\right)^0 = 1$ $a^0 = 1$

b. $\frac{t^3s^0}{t}$

$\frac{t^3s^0}{t} = \frac{t^3(1)}{t}$ $a^0 = 1$

$= \frac{t^3}{t}$ Simplify.

$= t^2$ Quotient of Powers

To investigate the meaning of a negative exponent, we can simplify expressions like $\frac{8^2}{8^5}$ in two ways.

Method 1

$\frac{8^2}{8^5} = 8^{2-5}$ Quotient of Powers

$= 8^{-3}$ Subtract.

Method 2

$\frac{8^2}{8^5} = \frac{\overset{1}{\cancel{8}} \cdot \overset{1}{\cancel{8}}}{\underset{1}{\cancel{8}} \cdot \underset{1}{\cancel{8}} \cdot 8 \cdot 8 \cdot 8}$ Definition of powers

$= \frac{1}{8^3}$ Simplify.

Since $\frac{8^2}{8^5}$ cannot have two different values, we can conclude that $8^{-3} = \frac{1}{8^3}$.

Key Concept — Negative Exponent

- **Words** For any nonzero number a and any integer n, a^{-n} is the reciprocal of a^n. In addition, the reciprocal of a^{-n} is a^n.
- **Symbols** For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.
- **Examples** $5^{-2} = \frac{1}{5^2}$ or $\frac{1}{25}$ $\quad \frac{1}{m^{-3}} = m^3$

www.algebra1.com/extra_examples

An expression involving exponents is not considered simplified if the expression contains negative exponents.

Example 4 Negative Exponents

Simplify each expression. Assume that no denominator is equal to zero.

a. $\frac{b^{-3}c^2}{d^{-5}}$

$$\frac{b^{-3}c^2}{d^{-5}} = \left(\frac{b^{-3}}{1}\right)\left(\frac{c^2}{1}\right)\left(\frac{1}{d^{-5}}\right)$$ Write as a product of fractions.

$$= \left(\frac{1}{b^3}\right)\left(\frac{c^2}{1}\right)\left(\frac{d^5}{1}\right)$$ $a^{-n} = \frac{1}{a^n}$

$$= \frac{c^2d^5}{b^3}$$ Multiply fractions.

b. $\frac{-3a^{-4}b^7}{21a^2b^7c^{-5}}$

$$\frac{-3a^{-4}b^7}{21a^2b^7c^{-5}} = \left(\frac{-3}{21}\right)\left(\frac{a^{-4}}{a^2}\right)\left(\frac{b^7}{b^7}\right)\left(\frac{1}{c^{-5}}\right)$$ Group powers with the same base.

$$= \frac{-1}{7}(a^{-4-2})(b^{7-7})(c^5)$$ Quotient of Powers and Negative Exponent Properties

$$= \frac{-1}{7}a^{-6}b^0c^5$$ Simplify.

$$= \frac{-1}{7}\left(\frac{1}{a^6}\right)(1)c^5$$ Negative Exponent and Zero Exponent Properties

$$= -\frac{c^5}{7a^6}$$ Multiply fractions.

Study Tip

Common Misconception

Do not confuse a negative number with a number raised to a negative power.

$3^{-1} = \frac{1}{3}$ $\quad -3 \neq \frac{1}{3}$

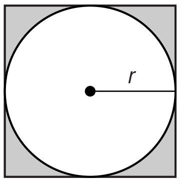
Standardized Test Practice

Example 5 Apply Properties of Exponents

Multiple-Choice Test Item

Write the ratio of the area of the circle to the area of the square in simplest form.

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{2\pi}{1}$ (D) $\frac{\pi}{3}$



Read the Test Item

A ratio is a comparison of two quantities. It can be written in fraction form.

Solve the Test Item

- area of circle $= \pi r^2$
  length of square = diameter of circle or $2r$
  area of square $= (2r)^2$
- $$\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{(2r)^2}$$ Substitute.

  $$= \frac{\pi}{4}r^{2-2}$$ Quotient of Powers

  $$= \frac{\pi}{4}r^0 \text{ or } \frac{\pi}{4}$$ $r^0 = 1$

The answer is B.

The Princeton Review

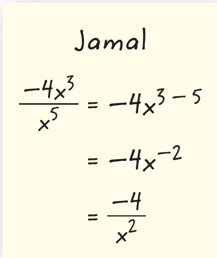
Test-Taking Tip

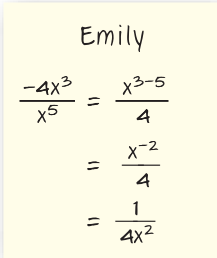
Some problems can be solved using estimation. The area of the circle is less than the area of the square. Therefore, the ratio of the two areas must be less than 1. Use 3 as an approximate value for π to determine which of the choices is less than 1.

Check for Understanding

Concept Check

1. **OPEN ENDED** Name two monomials whose product is $54x^2y^3$.
2. **Show** a method of simplifying $\frac{a^3b^5}{ab^2}$ using negative exponents instead of the Quotient of Powers Property.
3. **FIND THE ERROR** Jamal and Emily are simplifying $\frac{-4x^3}{x^5}$.





Who is correct? Explain your reasoning.

Guided Practice

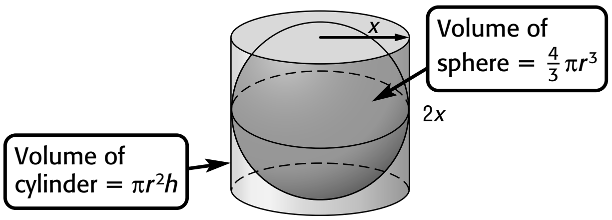
Simplify. Assume that no denominator is equal to zero.

4. $\frac{7^8}{7^2}$
5. $\frac{x^8y^{12}}{x^2y^7}$
6. $\left(\frac{2c^3d}{7z^2}\right)^3$
7. $y^0(y^5)(y^{-9})$
8. 13^{-2}
9. $\frac{c^{-5}}{d^3g^{-8}}$
10. $\frac{-5pq^7}{10p^6q^3}$
11. $\frac{(cd^{-2})^3}{(c^4d^9)^{-2}}$
12. $\frac{(4m^{-3}n^5)^0}{mn}$

Standardized Test Practice

13. Find the ratio of the volume of the cylinder to the volume of the sphere.

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{3\pi}{2}$



Practice and Apply

Homework Help

For Exercises	See Examples
14–21	1, 2
22–37	1–4

Extra Practice

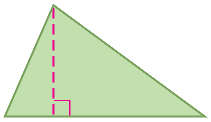
See page 837.

Simplify. Assume that no denominator is equal to zero.

14. $\frac{4^{12}}{4^2}$
15. $\frac{3^{13}}{3^7}$
16. $\frac{p^7n^3}{p^4n^2}$
17. $\frac{y^3z^9}{yz^2}$
18. $\left(\frac{5b^4n}{2a^6}\right)^2$
19. $\left(\frac{3m^7}{4x^5y^3}\right)^4$
20. $\frac{-2a^3}{10a^8}$
21. $\frac{15b}{45b^5}$
22. $x^3y^0x^{-7}$
23. $n^2(p^{-4})(n^{-5})$
24. 6^{-2}
25. 5^{-3}
26. $\left(\frac{4}{5}\right)^{-2}$
27. $\left(\frac{3}{2}\right)^{-3}$
28. $\frac{28a^7c^{-4}}{7a^3b^0c^{-8}}$
29. $\frac{30h^{-2}k^{14}}{5hk^{-3}}$
30. $\frac{18x^3y^4z^7}{-2x^2yz}$
31. $\frac{-19y^0z^4}{-3z^{16}}$
32. $\frac{(5r^{-2})^{-2}}{(2r^3)^2}$
33. $\frac{p^{-4}q^{-3}}{(p^5q^2)^{-1}}$
34. $\left(\frac{r^{-2}t^5}{t^{-1}}\right)^0$
35. $\left(\frac{4c^{-2}d}{b^{-2}c^3d^{-1}}\right)^0$
36. $\left(\frac{5b^{-2}n^4}{n^2z^{-3}}\right)^{-1}$
37. $\left(\frac{2a^{-2}bc^{-1}}{3ab^{-2}}\right)^{-3}$

38. The area of the rectangle is $24x^5y^3$ square units. Find the length of the rectangle.

39. The area of the triangle is $100a^3b$ square units. Find the height of the triangle.



SOUND **For Exercises 40–42, use the following information.**
The intensity of sound can be measured in watts per square meter. The table gives the watts per square meter for some common sounds.

Watts/Square Meter	Common Sounds
10^2	jet plane (30 m away)
10^1	pain level
10^0	amplified music (2 m away)
10^{-2}	noisy kitchen
10^{-3}	heavy traffic
10^{-6}	normal conversation
10^{-7}	average home
10^{-9}	soft whisper
10^{-12}	barely audible

40. How many times more intense is the sound from heavy traffic than the sound from normal conversation?

41. What sound is 10,000 times as loud as a noisy kitchen?

42. How does the intensity of a whisper compare to that of normal conversation?

More About. . .

Sound

Timbre is the quality of the sound produced by a musical instrument. Sound quality is what distinguishes the sound of a note played on a flute from the sound of the same note played on a trumpet with the same frequency and intensity.

Source: www.school.discovery.com

PROBABILITY **For Exercises 43 and 44, use the following information.**
If you toss a coin, the probability of getting heads is $\frac{1}{2}$. If you toss a coin 2 times, the probability of getting heads each time is $\frac{1}{2} \cdot \frac{1}{2}$ or $\left(\frac{1}{2}\right)^2$.

43. Write an expression to represent the probability of tossing a coin n times and getting n heads.

44. Express your answer to Exercise 43 as a power of 2.

LIGHT **For Exercises 45 and 46, use the table below.**

45. Express the range of the wavelengths of visible light using positive exponents. Then evaluate each expression.

46. Express the range of the wavelengths of X-rays using positive exponents. Then evaluate each expression.

Spectrum of Electromagnetic Radiation

Region	Wavelength (cm)
Radio	greater than 10
Microwave	10^1 to 10^{-2}
Infrared	10^{-2} to 10^{-5}
Visible	10^{-5} to 10^{-4}
Ultraviolet	10^{-4} to 10^{-7}
X-rays	10^{-7} to 10^{-9}
Gamma Rays	less than 10^{-9}

CRITICAL THINKING **Simplify. Assume that no denominator is equal to zero.**

47. $a^n(a^3)$

48. $(5^{4x-3})(5^{2x+1})$

49. $\frac{c^{x+7}}{c^{x-4}}$

50. $\frac{3b^{2n-9}}{b^{3(n-3)}}$

51. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How can you compare pH levels?

Include the following in your answer:
- an example comparing two pH levels using the properties of exponents.

Standardized Test Practice

52. What is the value of $\frac{2^2 \cdot 2^3}{2^{-2} \cdot 2^{-3}}$?

Ⓐ 2^{10} Ⓑ 2^{12} Ⓒ -1 Ⓓ $\frac{1}{2}$

53. **EXTENDED RESPONSE** Write a convincing argument to show why $3^0 = 1$ using the following pattern.
$3^5 = 243, 3^4 = 81, 3^3 = 27, 3^2 = 9, \ldots$

Maintain Your Skills

Mixed Review

Simplify. *(Lesson 8-1)*

54. $(m^3n)(mn^2)$

55. $(3x^4y^3)(4x^4y)$

56. $(a^3x^2)^4$

57. $(3cd^5)^2$

58. $[(2^3)^2]^2$

59. $(-3ab)^3(2b^3)^2$

NUTRITION **For Exercises 60 and 61, use the following information.**
Between the ages of 11 and 18, you should get at least 1200 milligrams of calcium each day. One ounce of mozzarella cheese has 147 milligrams of calcium, and one ounce of Swiss cheese has 219 milligrams. Suppose you wanted to eat no more than 8 ounces of cheese. *(Lesson 7-5)*

60. Draw a graph showing the possible amounts of each type of cheese you can eat and still get your daily requirement of calcium. Let x be the amount of mozzarella cheese and y be the amount of Swiss cheese.

61. List three possible solutions.

Write an equation of the line with the given slope and y-intercept. *(Lesson 5-3)*

62. slope: 1, y-intercept: -4

63. slope: -2, y-intercept: 3

64. slope: $-\frac{1}{3}$, y-intercept: -1

65. slope: $\frac{3}{2}$, y-intercept: 2

Graph each equation by finding the x- and y-intercepts. *(Lesson 4-5)*

66. $2y = x + 10$

67. $4x - y = 12$

68. $2x = 7 - 3y$

Find each square root. If necessary, round to the nearest hundredth. *(Lesson 2-7)*

69. $\pm\sqrt{121}$

70. $\sqrt{3.24}$

71. $-\sqrt{52}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review **Products of Powers**, see Lesson 8-1.)*

72. $10^2 \times 10^3$

73. $10^{-8} \times 10^{-5}$

74. $10^{-6} \times 10^9$

75. $10^8 \times 10^{-1}$

76. $10^4 \times 10^{-4}$

77. $10^{-12} \times 10$

Reading Mathematics

Mathematical Prefixes and Everyday Prefixes

You may have noticed that many prefixes used in mathematics are also used in everyday language. You can use the everyday meaning of these prefixes to better understand their mathematical meaning. The table shows two mathematical prefixes along with their meaning and an example of an everyday word using that prefix.

Prefix	Everyday Meaning	Example
mono-	**1.** one; single; alone	**monologue** A continuous series of jokes or comic stories delivered by one comedian.
bi-	**1.** two **2.** both **3.** both sides, parts, or directions	**bicycle** A vehicle consisting of a light frame mounted on two wire-spoked wheels one behind the other and having a seat, handlebars for steering, brakes, and two pedals or a small motor by which it is driven.
tri-	**1.** three **2.** occurring at intervals of three **3.** occurring three times during	**trilogy** A group of three dramatic or literary works related in subject or theme.
poly-	**1.** more than one; many; much	**polygon** A closed plane figure bounded by three or more line segments.

Source: *The American Heritage Dictionary of the English Language*

You can use your everyday understanding of prefixes to help you understand mathematical terms that use those prefixes.

Reading to Learn

1. Give an example of a geometry term that uses one of these prefixes. Then define that term.

2. **MAKE A CONJECTURE** Given your knowledge of the meaning of the word monomial, make a conjecture as to the meaning of each of the following mathematical terms.

   a. binomial **b.** trinomial **c.** polynomial

3. Research the following prefixes and their meanings.

   a. semi- **b.** hexa- **c.** octa-

8-3 Scientific Notation

What You'll Learn

- Express numbers in scientific notation and standard notation.
- Find products and quotients of numbers expressed in scientific notation.

Vocabulary

- scientific notation

Why is scientific notation important in astronomy?

Astronomers often work with very large numbers, such as the masses of planets. The mass of each planet in our solar system is given in the table. Notice that each value is written as the product of a number and a power of 10. These values are written in scientific notation.

Planet	Mass (kilograms)
Mercury	3.30×10^{23}
Venus	4.87×10^{24}
Earth	5.97×10^{24}
Mars	6.42×10^{23}
Jupiter	1.90×10^{27}
Saturn	5.69×10^{26}
Uranus	8.68×10^{25}
Neptune	1.02×10^{26}
Pluto	1.27×10^{22}

Source: NASA

SCIENTIFIC NOTATION When dealing with very large or very small numbers, keeping track of place value can be difficult. For this reason, numbers such as these are often expressed in **scientific notation**.

Key Concept — *Scientific Notation*

- **Words** A number is expressed in scientific notation when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.
- **Symbols** A number in scientific notation is written as $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Study Tip

Reading Math

Standard notation is the way in which you are used to seeing a number written, where the decimal point determines the place value for each digit of the number.

The following examples show one way of expressing a number that is written in scientific notation in its decimal or standard notation. Look for a relationship between the power of 10 and the position of the decimal point in the standard notation of the number.

$$6.59 \times 10^4 = 6.59 \times 10{,}000$$
$$= 65{,}900$$

The decimal point moved 4 places to the right.

$$4.81 \times 10^{-6} = 4.81 \times \frac{1}{10^6}$$
$$= 4.81 \times 0.000001$$
$$= 0.00000481$$

The decimal point moved 6 places to the left.

These examples suggest the following rule for expressing a number written in scientific notation in standard notation.

Concept Summary — Scientific to Standard Notation

Use these steps to express a number of the form $a \times 10^n$ in standard notation.

1. Determine whether $n > 0$ or $n < 0$.
2. If $n > 0$, move the decimal point in a to the right n places.
   If $n < 0$, move the decimal point in a to the left n places.
3. Add zeros, decimal point, and/or commas as needed to indicate place value.

Example 1 Scientific to Standard Notation

Express each number in standard notation.

a. $\mathbf{2.45 \times 10^8}$

$2.45 \times 10^8 = 245{,}000{,}000$ — $n = 8$; move decimal point 8 places to the right.

b. $\mathbf{3 \times 10^{-5}}$

$3 \times 10^{-5} = 0.00003$ — $n = -5$; move decimal point 5 places to the left.

To express a number in scientific notation, reverse the process used above.

Concept Summary — Standard to Scientific Notation

Use these steps to express a number in scientific notation.

1. Move the decimal point so that it is to the right of the first nonzero digit. The result is a decimal number a.
2. Observe the number of places n and the direction in which you moved the decimal point.
3. If the decimal point moved to the left, write as $a \times 10^n$.
   If the decimal point moved to the right, write as $a \times 10^{-n}$.

Example 2 Standard to Scientific Notation

Express each number in scientific notation.

a. 30,500,000

$30{,}500{,}000 \rightarrow 3.0500000 \times 10^n$ — Move decimal point 7 places to the left.

$30{,}500{,}000 = 3.05 \times 10^7$ — $a = 3.05$ and $n = 7$

b. 0.000781

$0.000781 \rightarrow 00007.81 \times 10^n$ — Move decimal point 4 places to the right.

$0.000781 = 7.81 \times 10^{-4}$ — $a = 7.81$ and $n = -4$

Study Tip

Scientific Notation

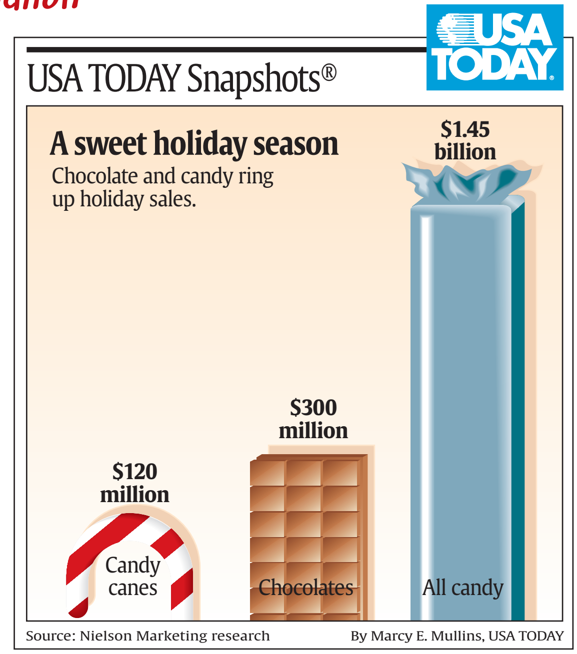
Notice that when a number is in scientific notation, no more than one digit is to the left of the decimal point.

You will often see large numbers in the media written using a combination of a number and a word, such as 3.2 million. To write this number in standard notation, rewrite the word *million* as 10^6. The exponent 6 indicates that the decimal point should be moved 6 places to the right.

$$3.2 \text{ million} = 3{,}200{,}000$$

Example 3 Use Scientific Notation

The graph shows chocolate and candy sales during a recent holiday season.



a. Express the sales of candy canes, chocolates, and all candy in standard notation.

Candy canes:
\$120 million = \$120,000,000

Chocolates:
\$300 million = \$300,000,000

All candy:
\$1.45 billion = \$1,450,000,000

b. Write each of these sales figures in scientific notation.

Candy canes:
$\$120{,}000{,}000 = \$1.2 \times 10^8$

Chocolates:
$\$300{,}000{,}000 = \$3.0 \times 10^8$

All candy: $\$1{,}450{,}000{,}000 = \$1.45 \times 10^9$

Log on for:
- Updated data
- More activities on scientific notation

www.algebra1.com/usa_today

PRODUCTS AND QUOTIENTS WITH SCIENTIFIC NOTATION

You can use scientific notation to simplify computation with very large and/or very small numbers.

Example 4 Multiplication with Scientific Notation

Evaluate $(5 \times 10^{-8})(2.9 \times 10^2)$. Express the result in scientific and standard notation.

$(5 \times 10^{-8})(2.9 \times 10^2)$

$= (5 \times 2.9)(10^{-8} \times 10^2)$ Commutative and Associative Properties

$= 14.5 \times 10^{-6}$ Product of Powers

$= (1.45 \times 10^1) \times 10^{-6}$ $14.5 = 1.45 \times 10^1$

$= 1.45 \times (10^1 \times 10^{-6})$ Associative Property

$= 1.45 \times 10^{-5}$ or 0.0000145 Product of Powers

Example 5 Division with Scientific Notation

Evaluate $\dfrac{1.2789 \times 10^9}{5.22 \times 10^5}$. Express the result in scientific and standard notation.

$\dfrac{1.2789 \times 10^9}{5.22 \times 10^5} = \left(\dfrac{1.2789}{5.22}\right)\left(\dfrac{10^9}{10^5}\right)$ Associative Property

$= 0.245 \times 10^4$ Quotient of Powers

$= (2.45 \times 10^{-1}) \times 10^4$ $0.245 = 2.45 \times 10^{-1}$

$= 2.45 \times (10^{-1} \times 10^4)$ Associative Property

$= 2.45 \times 10^3$ or 2450 Product of Powers

Check for Understanding

Concept Check

1. **Explain** how you know to use a positive or a negative exponent when writing a number in scientific notation.
2. **State** whether 65.2×10^3 is in scientific notation. Explain your reasoning.
3. **OPEN ENDED** Give an example of a large number written using a decimal number and a word. Write this number in standard and then in scientific notation.

Guided Practice

Express each number in standard notation.

4. 2×10^{-8}
5. 4.59×10^3
6. 7.183×10^{14}
7. 3.6×10^{-5}

Express each number in scientific notation.

8. 56,700,000
9. 0.00567
10. 0.00000000004
11. 3,002,000,000,000,000

Evaluate. Express each result in scientific and standard notation.

12. $(5.3 \times 10^2)(4.1 \times 10^5)$
13. $(2 \times 10^{-5})(9.4 \times 10^{-3})$
14. $\frac{1.5 \times 10^2}{2.5 \times 10^{12}}$
15. $\frac{1.25 \times 10^4}{2.5 \times 10^{-6}}$

Application

CREDIT CARDS **For Exercises 16 and 17, use the following information.**
During the year 2000, 1.65 billion credit cards were in use in the United States. During that same year, \$1.54 trillion was charged to these cards. (*Hint*: 1 trillion $= 1 \times 10^{12}$) **Source:** U.S. Department of Commerce

16. Express each of these values in standard and then in scientific notation.
17. Find the average amount charged per credit card.

Practice and Apply

Homework Help

For Exercises	See Examples
18–29	1
30–43	2
44–55	3, 4
56–59	5

Extra Practice

See page 837.

Express each number in standard notation.

18. 5×10^{-6}
19. 6.1×10^{-9}
20. 7.9×10^4
21. 8×10^7
22. 1.243×10^{-7}
23. 2.99×10^{-1}
24. 4.782×10^{13}
25. 6.89×10^0

PHYSICS **Express the number in each statement in standard notation.**

26. There are 2×10^{11} stars in the Andromeda Galaxy.
27. The center of the moon is 2.389×10^5 miles away from the center of Earth.
28. The mass of a proton is 1.67265×10^{-27} kilograms.
29. The mass of an electron is 9.1095×10^{-31} kilograms.

Express each number in scientific notation.

30. 50,400,000,000
31. 34,402,000
32. 0.000002
33. 0.00090465
34. 25.8
35. 380.7
36. 622×10^6
37. 87.3×10^{11}
38. 0.5×10^{-4}
39. 0.0081×10^{-3}
40. 94×10^{-7}
41. 0.001×10^{12}

WebQuest

The distances of the planets from the Sun can be written in scientific notation. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

42. **STARS** In the 1930s, the Indian physicist Subrahmanyan Chandrasekhar and others predicted the existence of neutron stars. These stars can have a density of 10 billion tons per teaspoonful. Express this density in scientific notation.

43. **PHYSICAL SCIENCE** The unit of measure for counting molecules is a *mole*. One mole of a substance is the amount that contains about 602,214,299,000,000,000,000,000 molecules. Write this number in scientific notation.

Evaluate. Express each result in scientific and standard notation.

44. $(8.9 \times 10^4)(4 \times 10^3)$
45. $(3 \times 10^6)(5.7 \times 10^2)$
46. $(5 \times 10^{-2})(8.6 \times 10^{-3})$
47. $(1.2 \times 10^{-5})(1.2 \times 10^{-3})$
48. $(3.5 \times 10^7)(6.1 \times 10^{-8})$
49. $(2.8 \times 10^{-2})(9.1 \times 10^6)$
50. $\dfrac{7.2 \times 10^9}{4.8 \times 10^4}$
51. $\dfrac{7.2 \times 10^3}{1.8 \times 10^7}$
52. $\dfrac{3.162 \times 10^{-4}}{5.1 \times 10^2}$
53. $\dfrac{1.035 \times 10^{-2}}{4.5 \times 10^3}$
54. $\dfrac{2.795 \times 10^{-8}}{4.3 \times 10^{-4}}$
55. $\dfrac{4.65 \times 10^{-1}}{5 \times 10^5}$

56. **HAIR GROWTH** The usual growth rate of human hair is 3.3×10^{-4} meter per day. If an individual hair grew for 10 years, how long would it be in meters? (Assume 365 days in a year.)

57. **NATIONAL DEBT** In April 2001, the national debt was about $5.745 trillion, and the estimated U.S. population was 283.9 million. About how much was each U.S. citizen's share of the national debt at that time?

Online Research **Data Update** What is the current U.S. population and amount of national debt? Visit www.algebra1.com/data_update to learn more.

More About. . .

Baseball

The contract Alex Rodriguez signed with the Texas Rangers on December 11, 2000, guarantees him $25.2 million a year for 10 seasons.

Source: Associated Press

58. **BASEBALL** The table below lists the greatest yearly salary for a major league baseball player for selected years.

Baseball Salary Milestones

Year	Player	Yearly Salary
1979	Nolan Ryan	$1 million
1982	George Foster	$2.04 million
1990	Jose Canseco	$4.7 million
1992	Ryne Sandberg	$7.1 million
1996	Ken Griffey, Jr.	$8.5 million
1997	Pedro Martinez	$12.5 million
2000	Alex Rodriguez	$25.2 million

Source: *USA TODAY*

About how many times as great was the yearly salary of Alex Rodriguez in 2000 as that of George Foster in 1982?

59. **ASTRONOMY** The Sun burns about 4.4×10^6 tons of hydrogen per second. How much hydrogen does the Sun burn in one year? (*Hint*: First, find the number of seconds in a year and write this number in scientific notation.)

60. **CRITICAL THINKING** Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

   a. If $1 \leq a < 10$ and n and p are integers, then $(a \times 10^n)^p = a^p \times 10^{np}$.

   b. The expression $a^p \times 10^{np}$ in part **a** is in scientific notation.

www.algebra1.com/self_check_quiz

61. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why is scientific notation important in astronomy?

Include the following in your answer:
- the mass of each of the planets in standard notation, and
- an explanation of how scientific notation makes presenting and computing with large numbers easier.

Standardized Test Practice

62. Which of the following is equivalent to 360×10^{-4}?

Ⓐ 3.6×10^{3} Ⓑ 3.6×10^{2} Ⓒ 3.6×10^{-2} Ⓓ 3.6×10^{-3}

63. **SHORT RESPONSE** There are an average of 25 billion red blood cells in the human body and about 270 million hemoglobin molecules in each red blood cell. Find the average number of hemoglobin molecules in the human body.

Graphing Calculator

SCIENTIFIC NOTATION You can use a graphing calculator to solve problems involving scientific notation. First, put your calculator in scientific mode. To enter 4.5×10^9, enter 4.5 [×] 10 [^] 9.

64. $(4.5 \times 10^9)(1.74 \times 10^{-2})$
65. $(7.1 \times 10^{-11})(1.2 \times 10^5)$
66. $(4.095 \times 10^5) \div (3.15 \times 10^8)$
67. $(6 \times 10^{-4}) \div (5.5 \times 10^{-7})$

Maintain Your Skills

Mixed Review

Simplify. Assume no denominator is equal to zero. *(Lesson 8-2)*

68. $\dfrac{49a^4b^7c^2}{7ab^4c^3}$
69. $\dfrac{-4n^3p^{-5}}{n^{-2}}$
70. $\dfrac{(8n^7)^2}{(3n^2)^{-3}}$

Determine whether each expression is a monomial. Write *yes* or *no*. *(Lesson 8-1)*

71. $3a + 4b$
72. $\dfrac{6}{n}$
73. $\dfrac{v^2}{3}$

Solve each inequality. Then check your solution and graph it on a number line. *(Lesson 6-1)*

74. $m - 3 < -17$
75. $-9 + d > 9$
76. $-x - 11 \geq 23$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression when $a = 5$, $b = -2$, and $c = 3$.**
*(To review **evaluating expressions**, see Lesson 1-2.)*

77. $5b^2$
78. $c^2 - 9$
79. $b^3 + 3ac$
80. $a^2 + 2a - 1$
81. $-2b^4 - 5b^3 - b$
82. $3.2c^3 + 0.5c^2 - 5.2c$

Practice Quiz 1

Lessons 8-1 through 8-3

Simplify. *(Lesson 8-1)*

1. $n^3(n^4)(n)$
2. $4ad(3a^3d)$
3. $(-2w^3z^4)^3(-4wz^3)^2$

Simplify. Assume that no denominator is equal to zero. *(Lesson 8-2)*

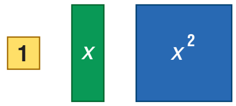
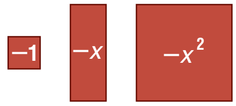
4. $\dfrac{25p^{10}}{15p^3}$
5. $\left(\dfrac{6k^3}{7np^4}\right)^2$
6. $\dfrac{4x^0y^2}{(3y^{-3}z^5)^{-2}}$

Evaluate. Express each result in scientific and standard notation. *(Lesson 8-3)*

7. $(6.4 \times 10^3)(7 \times 10^2)$
8. $(4 \times 10^2)(15 \times 10^{-6})$
9. $\dfrac{9.2 \times 10^3}{2.3 \times 10^5}$
10. $\dfrac{3.6 \times 10^7}{1.2 \times 10^{-2}}$

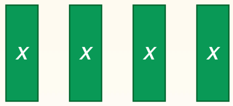
Polynomials

Algebra tiles can be used to model polynomials. A polynomial is a monomial or the sum of monomials. The diagram at the right shows the models.

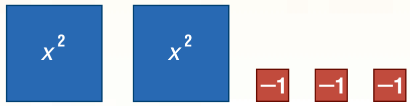
Polynomial Models	
Polynomials are modeled using three types of tiles.	
Each tile has an opposite.	

Use algebra tiles to model each polynomial.

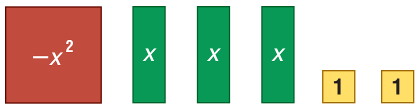
- **$4x$**
  To model this polynomial, you will need 4 green x tiles.



- **$2x^2 - 3$**
  To model this polynomial, you will need 2 blue x^2 tiles and 3 red -1 tiles.



- **$-x^2 + 3x + 2$**
  To model this polynomial, you will need 1 red $-x^2$ tile, 3 green x tiles, and 2 yellow 1 tiles.

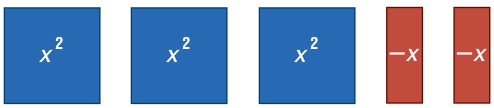


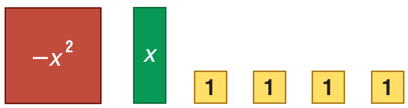
Model and Analyze

Use algebra tiles to model each polynomial. Then draw a diagram of your model.

1. $-2x^2$ **2.** $5x - 4$ **3.** $3x^2 - x$ **4.** $x^2 + 4x + 3$

Write an algebraic expression for each model.

5.


6.


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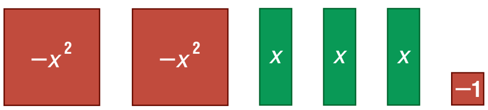
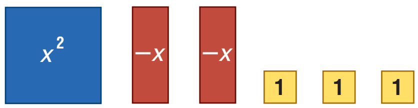


8.


9. **MAKE A CONJECTURE** Write a sentence or two explaining why algebra tiles are sometimes called *area tiles*.

8-4 Polynomials

What You'll Learn

- Find the degree of a polynomial.
- Arrange the terms of a polynomial in ascending or descending order.

Vocabulary

- polynomial
- binomial
- trinomial
- degree of a monomial
- degree of a polynomial

How are polynomials useful in modeling data?

The number of hours H spent per person per year playing video games from 1992 through 1997 is shown in the table. These data can be modeled by the equation

$$H = \frac{1}{4}(t^4 - 9t^3 + 26t^2 - 18t + 76),$$

where t is the number of years since 1992. The expression $t^4 - 9t^3 + 26t^2 - 18t + 76$ is an example of a polynomial.

Video Game Usage

Year	Hours spent per person
1992	19
1993	19
1994	22
1995	24
1996	26
1997	36

Source: U.S. Census Bureau

Study Tip

Common Misconception

Before deciding if an expression is a polynomial, write each term of the expression so that there are no variables in the denominator. Then look for negative exponents. Recall that the exponents of a monomial must be nonnegative integers.

DEGREE OF A POLYNOMIAL A **polynomial** is a monomial or a sum of monomials. Some polynomials have special names. A **binomial** is the sum of *two* monomials, and a **trinomial** is the sum of *three* monomials. Polynomials with more than three terms have no special names.

Monomial	Binomial	Trinomial
7	$3 + 4y$	$x + y + z$
$13n$	$2a + 3c$	$p^2 + 5p + 4$
$-5z^3$	$6x^2 + 3xy$	$a^2 - 2ab - b^2$
$4ab^3c^2$	$7pqr + pq^2$	$3v^2 - 2w + ab^3$

Example 1 Identify Polynomials

State whether each expression is a polynomial. If it is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*.

	Expression	Polynomial?	Monomial, Binomial, or Trinomial?
a.	$2x - 3yz$	Yes, $2x - 3yz = 2x + (-3yz)$. The expression is the sum of two monomials.	binomial
b.	$8n^3 + 5n^{-2}$	No. $5n^{-2} = \frac{5}{n^2}$, which is not a monomial.	none of these
c.	-8	Yes. -8 is a real number.	monomial
d.	$4a^2 + 5a + a + 9$	Yes. The expression simplifies to $4a^2 + 6a + 9$, so it is the sum of three monomials.	trinomial

Study Tip

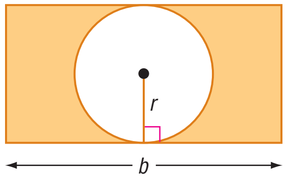
Like Terms

Be sure to combine any like terms before deciding if a polynomial is a monomial, binomial, or trinomial.

Polynomials can be used to express geometric relationships.

Example 2 Write a Polynomial

GEOMETRY **Write a polynomial to represent the area of the shaded region.**



Words The area of the shaded region is the area of the rectangle minus the area of the circle.

Variables area of shaded region = A
width of rectangle = $2r$
rectangle area = $b(2r)$
circle area = πr^2

Equation

area of shaded region	=	rectangle area	−	circle area
A	=	$b(2r)$	−	πr^2
A	=	$2br$	−	πr^2

The polynomial representing the area of the shaded region is $2br - \pi r^2$.

The **degree of a monomial** is the sum of the exponents of all its variables.

The **degree of a polynomial** is the greatest degree of any term in the polynomial. To find the degree of a polynomial, you must find the degree of each term.

Monomial	Degree
$8y^4$	4
$3a$	1
$-2xy^2z^3$	$1 + 2 + 3$ or 6
7	0

Example 3 Degree of a Polynomial

Find the degree of each polynomial.

	Polynomial	Terms	Degree of Each Term	Degree of Polynomial
a.	$5mn^2$	$5mn^2$	1, 2	3
b.	$-4x^2y^2 + 3x^2 + 5$	$-4x^2y^2$, $3x^2$, 5	4, 2, 0	4
c.	$3a + 7ab - 2a^2b + 16$	$3a$, $7ab$, $2a^2b$, 16	1, 2, 3, 0	3

Study Tip

Degrees of 1 and 0
- Since $a = a^1$, the monomial $3a$ can be rewritten as $3a^1$. Thus $3a$ has degree 1.
- Since $x^0 = 1$, the monomial 7 can be rewritten as $7x^0$. Thus 7 has degree 0.

WRITE POLYNOMIALS IN ORDER The terms of a polynomial are usually arranged so that the powers of one variable are in *ascending* (increasing) order or *descending* (decreasing) order.

Example 4 Arrange Polynomials in Ascending Order

Arrange the terms of each polynomial so that the powers of x are in ascending order.

a. $7x^2 + 2x^4 - 11$

$7x^2 + 2x^4 - 11 = 7x^2 + 2x^4 - 11x^0$ $\quad x^0 = 1$

$= -11 + 7x^2 + 2x^4$ $\quad$ Compare powers of x: $0 < 2 < 4$.

b. $2xy^3 + y^2 + 5x^3 - 3x^2y$

$2xy^3 + y^2 + 5x^3 - 3x^2y$

$= 2x^1y^3 + y^2 + 5x^3 - 3x^2y^1$ $\quad x = x^1$

$= y^2 + 2xy^3 - 3x^2y + 5x^3$ $\quad$ Compare powers of x: $0 < 1 < 2 < 3$.

www.algebra1.com/extra_examples

Example 5 Arrange Polynomials in Descending Order

Arrange the terms of each polynomial so that the powers of x are in descending order.

a. $6x^2 + 5 - 8x - 2x^3$

$6x^2 + 5 - 8x - 2x^3 = 6x^2 + 5x^0 - 8x^1 - 2x^3$ $x^0 = 1$ and $x = x^1$

$= -2x^3 + 6x^2 - 8x + 5$ $3 > 2 > 1 > 0$

b. $3a^3x^2 - a^4 + 4ax^5 + 9a^2x$

$3a^3x^2 - a^4 + 4ax^5 + 9a^2x = 3a^3x^2 - a^4x^0 + 4a^1x^5 + 9a^2x^1$ $a = a^1$, $x^0 = 1$, and $x = x^1$

$= 4ax^5 + 3a^3x^2 + 9a^2x - a^4$ $5 > 2 > 1 > 0$

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of a monomial of degree zero.
2. **Explain** why a polynomial cannot contain a variable raised to a negative power.
3. **Determine** whether each statement is *true* or *false*. If false, give a counterexample.
   a. All binomials are polynomials.
   b. All polynomials are monomials.
   c. All monomials are polynomials.

Guided Practice

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

4. $5x - 3xy + 2x$
5. $\frac{2z}{5}$
6. $9a^2 + 7a - 5$

Find the degree of each polynomial.

7. 1
8. $3x + 2$
9. $2x^2y^3 + 6x^4$

Arrange the terms of each polynomial so that the powers of x are in ascending order.

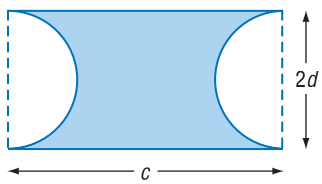
10. $6x^3 - 12 + 5x$
11. $-7a^2x^3 + 4x^2 - 2ax^5 + 2a$

Arrange the terms of each polynomial so that the powers of x are in descending order.

12. $2c^5 + 9cx^2 + 3x$
13. $y^3 + x^3 + 3x^2y + 3xy^2$

Application

14. **GEOMETRY** Write a polynomial to represent the area of the shaded region.



Practice and Apply

Homework Help

For Exercises	See Examples
15–20	1
21–24	2
25–36	3
37–52	4, 5

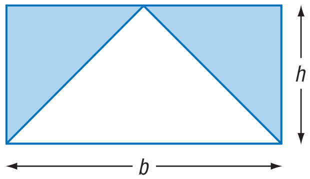
State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

15. 14
16. $\frac{6m^2}{p} + p^3$
17. $7b - 3.2c + 8b$
18. $\frac{1}{3}x^2 + x - 2$
19. $6gh^2 - 4g^2h + g$
20. $-4 + 2a + \frac{5}{a^2}$

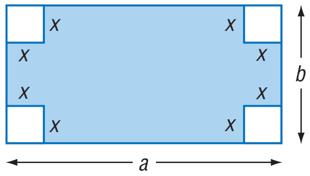
Extra Practice
See page 838.

GEOMETRY **Write a polynomial to represent the area of each shaded region.**

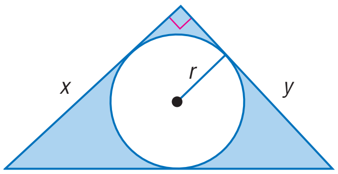
21.



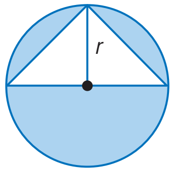
22.



23.



24.



Find the degree of each polynomial.

25. $5x^3$
26. $9y$
27. $4ab$
28. -13
29. $c^4 + 7c^2$
30. $6n^3 - n^2p^2$
31. $15 - 8ag$
32. $3a^2b^3c^4 - 18a^5c$
33. $2x^3 - 4y + 7xy$
34. $3z^5 - 2x^2y^3z - 4x^2z$
35. $7 + d^5 - b^2c^2d^3 + b^6$
36. $11r^2t^4 - 2s^4t^5 + 24$

Arrange the terms of each polynomial so that the powers of x are in ascending order.

37. $2x + 3x^2 - 1$
38. $9x^3 + 7 - 3x^5$
39. $c^2x^3 - c^3x^2 + 8c$
40. $x^3 + 4a + 5a^2x^6$
41. $4 + 3ax^5 + 2ax^2 - 5a^7$
42. $10x^3y^2 - 3x^9y + 5y^4 + 2x^2$
43. $3xy^2 - 4x^3 + x^2y + 6y$
44. $-8a^5x + 2ax^4 - 5 - a^2x^2$

Arrange the terms of each polynomial so that the powers of x are in descending order.

45. $5 + x^5 + 3x^3$
46. $2x - 1 + 6x^2$
47. $4a^3x^2 - 5a + 2a^2x^3$
48. $b^2 + x^2 - 2xb$
49. $c^2 + cx^3 - 5c^3x^2 + 11x$
50. $9x^2 + 3 + 4ax^3 - 2a^2x$
51. $8x - 9x^2y + 7y^2 - 2x^4$
52. $4x^3y + 3xy^4 - x^2y^3 + y^4$

53. MONEY Write a polynomial to represent the value of q quarters, d dimes, and n nickels.

54. MULTIPLE BIRTHS The number of quadruplet births Q in the United States from 1989 to 1998 can be modeled by $Q = -0.5t^3 + 11.7t^2 - 21.5t + 218.6$, where t represents the number of years since 1989. For what values of t does this model no longer give realistic data? Explain your reasoning.

More About. . .

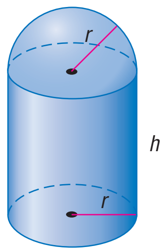
Multiple Births

From 1980 to 1997, the number of triplet and higher births rose 404% (from 1377 to 6737 births). This steep climb in multiple births coincides with the increased use of fertility drugs.

Source: National Center for Health and Statistics

PACKAGING **For Exercises 55 and 56, use the following information.**

A convenience store sells milkshakes in cups with semispherical lids. The volume of a cylinder is the product of π, the square of the radius r, and the height h. The volume of a sphere is the product of $\frac{4}{3}$, π, and the cube of the radius.



55. Write a polynomial that represents the volume of the container.

56. If the height of the container is 6 inches and the radius is 2 inches, find the volume of the container.

57. **CRITICAL THINKING** Tell whether the following statement is *true* or *false*. Explain your reasoning.
The degree of a binomial can never be zero.

58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are polynomials useful in modeling data?

Include the following in your answer:

- a discussion of the accuracy of the equation by evaluating the polynomial for $t = \{0, 1, 2, 3, 4, 5\}$, and
- an example of how and why someone might use this equation.

Standardized Test Practice

59. If $x = -1$, then $3x^3 + 2x^2 + x + 1 =$

Ⓐ -5. Ⓑ -1. Ⓒ 1. Ⓓ 2.

60. **QUANTITATIVE COMPARISON** **Compare the quantity in Column A and the quantity in Column B. Then determine whether:**

Ⓐ **the quantity in Column A is greater,**
Ⓑ **the quantity in Column B is greater,**
Ⓒ **the two quantities are equal, or**
Ⓓ **the relationship cannot be determined from the information given.**

Column A	Column B
the degree of $5x^2y^3$	the degree of $3x^3y^2$

Maintain Your Skills

Mixed Review

Express each number in scientific notation. *(Lesson 8-3)*

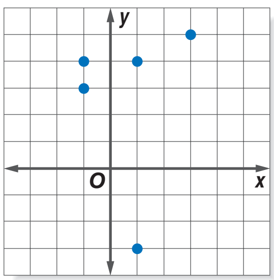
61. 12,300,000 62. 0.00345 63. 12×10^6 64. 0.77×10^{-10}

Simplify. Assume that no denominator is equal to zero. *(Lesson 8-2)*

65. $a^0b^{-2}c^{-1}$ 66. $\frac{-5n^5}{n^8}$ 67. $\left(\frac{4x^3y^2}{3z}\right)^2$ 68. $\frac{(-y)^5m^8}{y^3m^{-7}}$

Determine whether each relation is a function. *(Lesson 4-6)*

69.



70.

x	y
−2	−2
0	1
3	4
5	−2

71. **PROBABILITY** A card is selected at random from a standard deck of 52 cards. What is the probability of selecting a black card? *(Lesson 2-6)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify each expression. If not possible, write *simplified*.**
*(To review **evaluating expressions**, see Lesson 1-5.)*

72. $3n + 5n$ 73. $9a^2 + 3a - 2a^2$ 74. $12x^2 + 8x - 6$

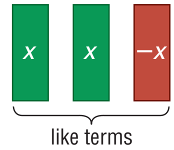
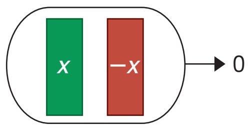
75. $-3a + 5b + 4a - 7b$ 76. $4x + 3y - 6 + 7x + 8 - 10y$

Algebra Activity

A Preview of Lesson 8-5

Adding and Subtracting Polynomials

Monomials such as $5x$ and $-3x$ are called *like terms* because they have the same variable to the same power. When you use algebra tiles, you can recognize like terms because the individual tiles have the same size and shape.

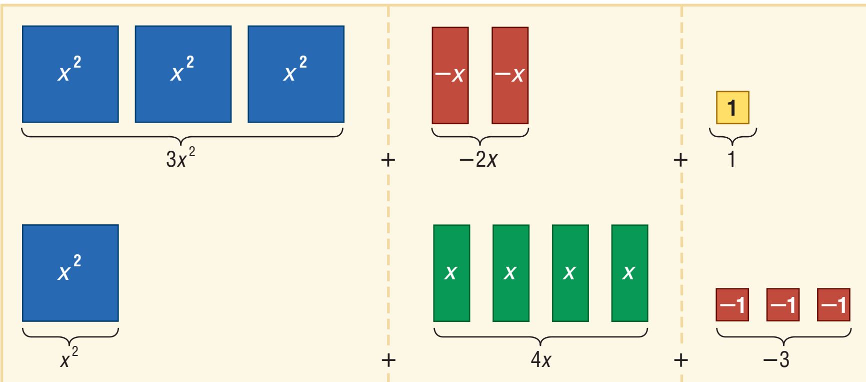
Polynomial Models	
Like terms are represented by tiles that have the same shape and size.	
A *zero pair* may be formed by pairing one tile with its opposite. You can remove or add zero pairs without changing the polynomial.	

Activity 1 Use algebra tiles to find $(3x^2 - 2x + 1) + (x^2 + 4x - 3)$.

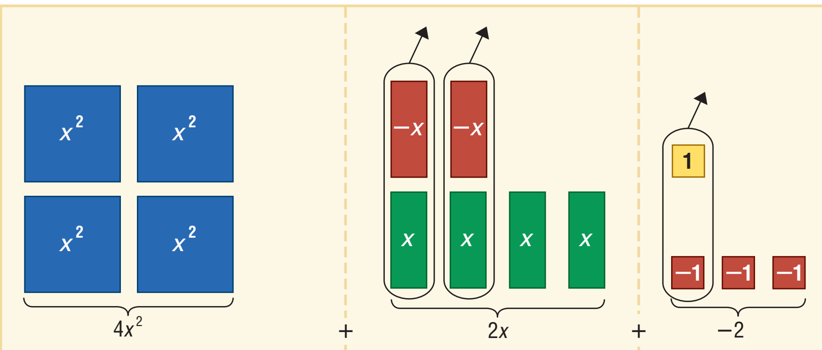
Step 1 Model each polynomial.

$3x^2 - 2x + 1 \rightarrow$

$x^2 + 4x - 3 \rightarrow$



Step 2 Combine like terms and remove zero pairs.

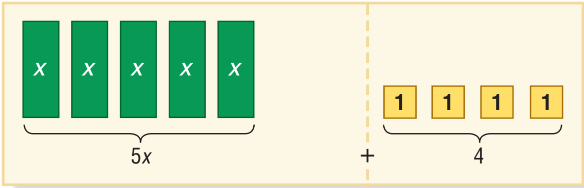


Step 3 Write the polynomial for the tiles that remain.

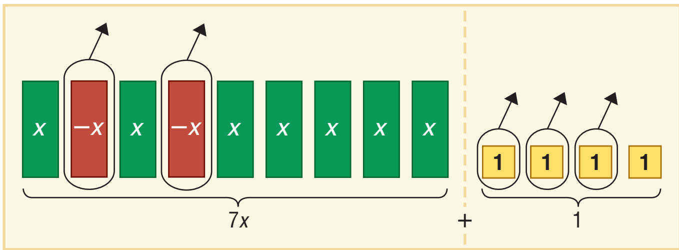
$(3x^2 - 2x + 1) + (x^2 + 4x - 3) = 4x^2 + 2x - 2$

Activity 2 **Use algebra tiles to find $(5x + 4) - (-2x + 3)$.**

Step 1 Model the polynomial $5x + 4$.



Step 2 To subtract $-2x + 3$, you must remove 2 red $-x$ tiles and 3 yellow 1 tiles. You can remove the yellow 1 tiles, but there are no red $-x$ tiles. Add 2 zero pairs of x tiles. Then remove the 2 red $-x$ tiles.



Step 3 Write the polynomial for the tiles that remain.
$(5x + 4) - (-2x + 3) = 7x + 1$

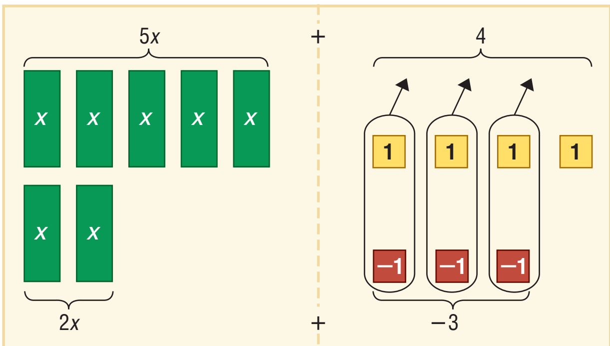
Recall that you can subtract a number by adding its additive inverse or opposite. Similarly, you can subtract a polynomial by adding its opposite.

Activity 3 **Use algebra tiles and the additive inverse, or opposite, to find $(5x + 4) - (-2x + 3)$.**

Step 1 To find the difference of $5x + 4$ and $-2x + 3$, add $5x + 4$ and the opposite of $-2x + 3$.

$5x + 4$ →

The opposite of $-2x + 3$ is $2x - 3$. →



Step 2 Write the polynomial for the tiles that remain.
$(5x + 4) - (-2x + 3) = 7x + 1$ Notice that this is the same answer as in Activity 2.

Model and Analyze

Use algebra tiles to find each sum or difference.

1. $(5x^2 + 3x - 4) + (2x^2 - 4x + 1)$
2. $(2x^2 + 5) + (3x^2 - 2x + 6)$
3. $(-4x^2 + x) + (5x - 2)$
4. $(3x^2 + 4x + 2) - (x^2 - 5x - 5)$
5. $(-x^2 + 7x) - (2x^2 + 3x)$
6. $(8x + 4) - (6x^2 + x - 3)$
7. Find $(2x^2 - 3x + 1) - (2x + 3)$ using each method from Activity 2 and Activity 3. Illustrate with drawings and explain in writing how zero pairs are used in each case.

8-5 Adding and Subtracting Polynomials

What You'll Learn

- Add polynomials.
- Subtract polynomials.

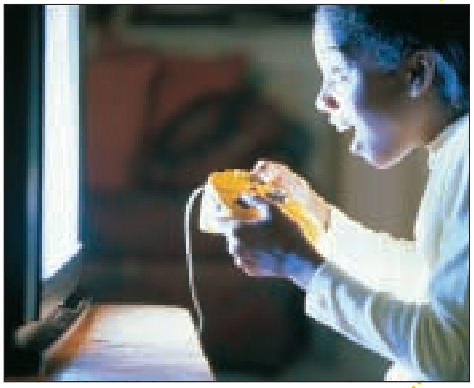
How can adding polynomials help you model sales?

From 1996 to 1999, the amount of sales (in billions of dollars) of video games V and traditional toys R in the United States can be modeled by the following equations, where t is the number of years since 1996.

Source: *Toy Industry Fact Book*

$$V = -0.05t^3 + 0.05t^2 + 1.4t + 3.6$$
$$R = 0.5t^3 - 1.9t^2 + 3t + 19$$

The total toy sales T is the sum of the video game sales V and traditional toy sales R.



ADD POLYNOMIALS To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms.

Example 1 Add Polynomials

Find $(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$.

Method 1 Horizontal

Group like terms together.

$(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$

$= [3x^2 + (-7x^2)] + (-4x + 2x) + [8 + (-5)]$ Associative and Commutative Properties

$= -4x^2 - 2x + 3$ Add like terms.

Method 2 Vertical

Align the like terms in columns and add.

$$\begin{array}{r} 3x^2 - 4x + 8 \\ (+)\ -7x^2 + 2x - 5 \\ \hline -4x^2 - 2x + 3 \end{array}$$

Notice that terms are in descending order with like terms aligned.

Study Tip

Adding Columns
When adding like terms in column form, remember that you are adding integers. Rewrite each monomial to eliminate subtractions. For example, you could rewrite $3x^2 - 4x + 8$ as $3x^2 + (-4x) + 8$.

SUBTRACT POLYNOMIALS Recall that you can subtract a rational number by adding its opposite or additive inverse. Similarly, you can subtract a polynomial by adding its additive inverse.

To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

Polynomial	Additive Inverse
$-5m + 3n$	$5m - 3n$
$2y^2 - 6y + 11$	$-2y^2 + 6y - 11$
$7a + 9b - 4$	$-7a - 9b + 4$

Example 2 Subtract Polynomials

Find $(3n^2 + 13n^3 + 5n) - (7n + 4n^3)$.

Method 1 Horizontal

Subtract $7n + 4n^3$ by adding its additive inverse.

$$\begin{aligned}
&(3n^2 + 13n^3 + 5n) - (7n + 4n^3) \\
&= (3n^2 + 13n^3 + 5n) + (-7n - 4n^3) && \text{The additive inverse of } 7n + 4n^3 \text{ is } -7n - 4n^3. \\
&= 3n^2 + [13n^3 + (-4n^3)] + [5n + (-7n)] && \text{Group like terms.} \\
&= 3n^2 + 9n^3 - 2n && \text{Add like terms.}
\end{aligned}$$

Method 2 Vertical

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rrr}
 & 3n^2 + 13n^3 + 5n \\
(-) & 4n^3 + 7n \\
\hline
\end{array}
\quad \text{Add the opposite.} \quad
\begin{array}{rr}
 & 3n^2 + 13n^3 + 5n \\
(+) & -4n^3 - 7n \\
\hline
 & 3n^2 + 9n^3 - 2n
\end{array}$$

Thus, $(3n^2 + 13n^3 + 5n) - (7n + 4n^3) = 3n^2 + 9n^3 - 2n$ or, arranged in descending order, $9n^3 + 3n^2 - 2n$.

Study Tip

Inverse of a Polynomial

When finding the additive inverse of a polynomial, remember to find the additive inverse of *every* term.

When polynomials are used to model real-world data, their sums and differences can have real-world meaning too.

Example 3 Subtract Polynomials

EDUCATION The total number of public school teachers T consists of two groups, elementary E and secondary S. From 1985 through 1998, the number (in thousands) of secondary teachers and total teachers in the United States could be modeled by the following equations, where n is the number of years since 1985.

$$S = 11n + 942$$
$$T = 44n + 2216$$

a. Find an equation that models the number of elementary teachers E for this time period.

You can find a model for E by subtracting the polynomial for S from the polynomial for T.

$$\begin{array}{lrr}
\text{Total} & & 44n + 2216 \\
-\ \text{Secondary} & (-) & 11n + 942 \\
\hline
\text{Elementary} & &
\end{array}
\quad \text{Add the opposite.} \quad
\begin{array}{rr}
 & 44n + 2216 \\
(+) & -11n - 942 \\
\hline
 & 33n + 1274
\end{array}$$

An equation is $E = 33n + 1274$.

b. Use the equation to predict the number of elementary teachers in the year 2010.

The year 2010 is 2010 − 1985 or 25 years after the year 1985.

If this trend continues, the number of elementary teachers in 2010 would be $33(25) + 1274$ thousand or about 2,099,000.

Career Choices

Teacher

The educational requirements for a teaching license vary by state. In 1999, the average public K–12 teacher salary was $40,582.

Online Research For information about a career as a teacher, visit: www.algebra1.com/careers

CONTENTS

Check for Understanding

Concept Check

1. **Explain** why $5xy^2$ and $3x^2y$ are *not* like terms.
2. **OPEN ENDED** Write two polynomials whose difference is $2x^2 + x + 3$.
3. **FIND THE ERROR** Esteban and Kendra are finding $(5a - 6b) - (2a + 5b)$.

Esteban	Kendra
$(5a - 6b) - (2a + 5b)$	$(5a - 6b) - (2a + 5b)$
$= (-5a + 6b) + (-2a - 5b)$	$= (5a - 6b) + (-2a - 5b)$
$= -7a + b$	$= 3a - 11b$

Who is correct? Explain your reasoning.

Guided Practice

Find each sum or difference.

4. $(4p^2 + 5p) + (-2p^2 + p)$
5. $(5y^2 - 3y + 8) + (4y^2 - 9)$
6. $(8cd - 3d + 4c) + (-6 + 2cd)$
7. $(6a^2 + 7a - 9) - (-5a^2 + a - 10)$
8. $(g^3 - 2g^2 + 5g + 6) - (g^2 + 2g)$
9. $(3ax^2 - 5x - 3a) - (6a - 8a^2x + 4x)$

Application

POPULATION **For Exercises 10 and 11, use the following information.**
From 1990 through 1999, the female population F and the male population M of the United States (in thousands) is modeled by the following equations, where n is the number of years since 1990. **Source:** U.S. Census Bureau

$$F = 1247n + 126{,}971 \qquad M = 1252n + 120{,}741$$

10. Find an equation that models the total population T in thousands of the United States for this time period.
11. If this trend continues, what will the population of the United States be in 2010?

Practice and Apply

Homework Help

For Exercises	See Examples
12–31	1, 2
32, 33	3

Extra Practice

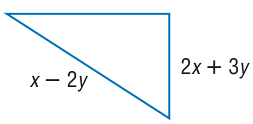
See page 838.

Find each sum or difference.

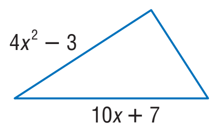
12. $(6n^2 - 4) + (-2n^2 + 9)$
13. $(9z - 3z^2) + (4z - 7z^2)$
14. $(3 + a^2 + 2a) + (a^2 - 8a + 5)$
15. $(-3n^2 - 8 + 2n) + (5n + 13 + n^2)$
16. $(x + 5) + (2y + 4x - 2)$
17. $(2b^3 - 4b + b^2) + (-9b^2 + 3b^3)$
18. $(11 + 4d^2) - (3 - 6d^2)$
19. $(4g^3 - 5g) - (2g^3 + 4g)$
20. $(-4y^3 - y + 10) - (4y^3 + 3y^2 - 7)$
21. $(4x + 5xy + 3y) - (3y + 6x + 8xy)$
22. $(3x^2 + 8x + 4) - (5x^2 - 4)$
23. $(5ab^2 + 3ab) - (2ab^2 + 4 - 8ab)$
24. $(x^3 - 7x + 4x^2 - 2) - (2x^2 - 9x + 4)$
25. $(5x^2 + 3a^2 - 5x) - (2x^2 - 5ax + 7x)$
26. $(3a + 2b - 7c) + (6b - 4a + 9c) + (-7c - 3a - 2b)$
27. $(5x^2 - 3) + (x^2 - x + 11) + (2x^2 - 5x + 7)$
28. $(3y^2 - 8) + (5y + 9) - (y^2 + 6y - 4)$
29. $(9x^3 + 3x - 13) - (6x^2 - 5x) + (2x^3 - x^2 - 8x + 4)$

GEOMETRY **The measures of two sides of a triangle are given. If P is the perimeter, find the measure of the third side.**

30. $P = 7x + 3y$



31. $P = 10x^2 - 5x + 16$



MOVIES For Exercises 32 and 33, use the following information.
From 1990 to 1999, the number of indoor movie screens I and total movie screens T in the U.S. could be modeled by the following equations, where n is the number of years since 1990.

$$I = 161.6n^2 - 20n + 23{,}326 \qquad T = 160.3n^2 - 26n + 24{,}226$$

32. Find an equation that models the number of outdoor movie screens D in the U.S. for this time period.

33. If this trend continues, how many outdoor movie screens will there be in the year 2010?

More About. . .

Movies

In 1998, attendance at movie theaters was at its highest point in 40 years with 1.48 billion tickets sold for a record $6.95 billion in gross income.

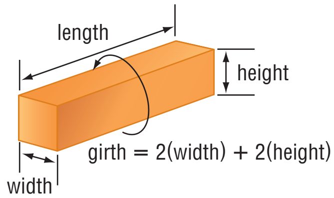
Source: The National Association of Theatre Owners

NUMBER TRICK For Exercises 34 and 35, use the following information.
Think of a two-digit number whose ones digit is greater than its tens digit. Multiply the difference of the two digits by 9 and add the result to your original number. Repeat this process for several other such numbers.

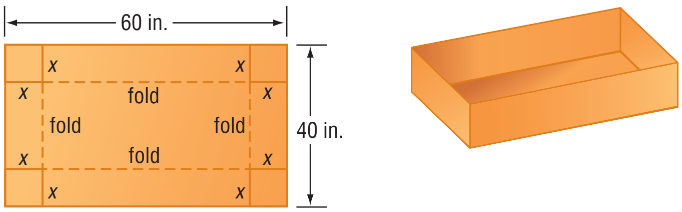
34. What observation can you make about your results?

35. Justify that your observation holds for all such two-digit numbers by letting x equal the tens digit and y equal the ones digit of the original number. (*Hint*: The original number is then represented by $10x + y$.)

POSTAL SERVICE For Exercises 36–40, use the information below and in the figure at the right.
The U.S. Postal Service restricts the sizes of boxes shipped by parcel post. The sum of the length and the girth of the box must not exceed 108 inches.



Suppose you want to make an open box using a 60-by-40 inch piece of cardboard by cutting squares out of each corner and folding up the flaps. The lid will be made from another piece of cardboard. You do not know how big the squares should be, so for now call the length of the side of each square x.



36. Write a polynomial to represent the length of the box formed.

37. Write a polynomial to represent the width of the box formed.

38. Write a polynomial to represent the girth of the box formed.

39. Write and solve an inequality to find the least possible value of x you could use in designing this box so it meets postal regulations.

40. What is the greatest integral value of x you could use to design this box if it does not have to meet regulations?

CRITICAL THINKING For Exercises 41–43, suppose x is an integer.

41. Write an expression for the next integer greater than x.

42. Show that the sum of two consecutive integers, x and the next integer after x, is always odd. (*Hint*: A number is considered even if it is divisible by 2.)

43. What is the least number of consecutive integers that must be added together to always arrive at an even integer?

CONTENTS

44. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can adding polynomials help you model sales?

Include the following in your answer:

- an equation that models total toy sales, and
- an example of how and why someone might use this equation.

Standardized Test Practice

45. The perimeter of the rectangle shown at the right is $16a + 2b$. Which of the following expressions represents the length of the rectangle?

(A) $3a + 2b$ (B) $10a + 2b$

(C) $2a - 3b$ (D) $6a + 4b$

46. If $a^2 - 2ab + b^2 = 36$ and $a^2 - 3ab + b^2 = 22$, find ab.

(A) 6 (B) 8 (C) 12 (D) 14

Maintain Your Skills

Mixed Review

Find the degree of each polynomial. *(Lesson 8-4)*

47. $15t^3y^2$ **48.** 24 **49.** $m^2 + n^3$ **50.** $4x^2y^3z - 5x^3z$

Express each number in standard notation. *(Lesson 8-3)*

51. 8×10^6 **52.** 2.9×10^5 **53.** 5×10^{-4} **54.** 4.8×10^{-7}

KEYBOARDING **For Exercises 55–59, use the table below that shows the keyboarding speeds and experience of 12 students.** *(Lesson 5-2)*

Experience (weeks)	4	7	8	1	6	3	5	2	9	6	7	10
Keyboarding Speed (wpm)	33	45	46	20	40	30	38	22	52	44	42	55

55. Make a scatter plot of these data.

56. Draw a best-fit line for the data.

57. Find the equation of the line.

58. Use the equation to predict the keyboarding speed of a student after a 12-week course.

59. Can this equation be used to predict the speed for any number of weeks of experience? Explain.

State the domain and range of each relation. *(Lesson 4-3)*

60. $\{(-2, 5), (0, -2), (-6, 3)\}$ **61.** $\{(-4, 2), (-1, -3), (5, 0), (-4, 1)\}$

62. **MODEL TRAINS** One of the most popular sizes of model trains is called the HO. Every dimension of the HO model measures $\frac{1}{87}$ times that of a real engine. The HO model of a modern diesel locomotive is about 8 inches long. About how many feet long is the real locomotive? *(Lesson 3-6)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review the **Distributive Property**, see Lesson 1-7.)*

63. $6(3x - 8)$ **64.** $-2(b + 9)$ **65.** $-7(-5p + 4q)$

66. $9(3a + 5b - c)$ **67.** $8(x^2 + 3x - 4)$ **68.** $-3(2a^2 - 5a + 7)$

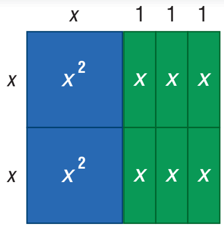
8-6 Multiplying a Polynomial by a Monomial

What You'll Learn

- Find the product of a monomial and a polynomial.
- Solve equations involving polynomials.

How is finding the product of a monomial and a polynomial related to finding the area of a rectangle?

The algebra tiles shown are grouped together to form a rectangle with a width of $2x$ and a length of $x + 3$. Notice that the rectangle consists of 2 blue x^2 tiles and 6 green x tiles. The area of the rectangle is the sum of these algebra tiles or $2x^2 + 6x$.



PRODUCT OF MONOMIAL AND POLYNOMIAL

The Distributive Property can be used to multiply a polynomial by a monomial.

Study Tip

Look Back
To review the **Distributive Property**, see Lesson 1-5.

Example 1 Multiply a Polynomial by a Monomial

Find $-2x^2(3x^2 - 7x + 10)$.

Method 1 Horizontal

$-2x^2(3x^2 - 7x + 10)$

$= -2x^2(3x^2) - (-2x^2)(7x) + (-2x^2)(10)$ Distributive Property

$= -6x^4 - (-14x^3) + (-20x^2)$ Multiply.

$= -6x^4 + 14x^3 - 20x^2$ Simplify.

Method 2 Vertical

$$\begin{array}{lr} & 3x^2 - 7x + 10 \\ (\times) & -2x^2 \\ \hline & -6x^4 + 14x^3 - 20x^2 \end{array}$$

Distributive Property

Multiply.

When expressions contain like terms, simplify by combining the like terms.

Example 2 Simplify Expressions

Simplify $4(3d^2 + 5d) - d(d^2 - 7d + 12)$.

$4(3d^2 + 5d) - d(d^2 - 7d + 12)$

$= 4(3d^2) + 4(5d) + (-d)(d^2) - (-d)(7d) + (-d)(12)$ Distributive Property

$= 12d^2 + 20d + (-d^3) - (-7d^2) + (-12d)$ Product of Powers

$= 12d^2 + 20d - d^3 + 7d^2 - 12d$ Simplify.

$= -d^3 + (12d^2 + 7d^2) + (20d - 12d)$ Commutative and Associative Properties

$= -d^3 + 19d^2 + 8d$ Combine like terms.

Example 3 Use Polynomial Models

PHONE SERVICE **Greg pays a fee of \$20 a month for local calls. Long-distance rates are 6¢ per minute for in-state calls and 5¢ per minute for out-of-state calls. Suppose Greg makes 300 minutes of long-distance phone calls in January and *m* of those minutes are for in-state calls.**

a. Find an expression for Greg's phone bill for January.

Words The bill is the sum of the monthly fee, in-state charges, and the out-of-state charges.

Variables If m = number of minutes of in-state calls, then $300 - m$ = number of minutes of out-of-state calls. Let B = phone bill for the month of January.

	bill	=	service fee	+	in-state minutes	·	6¢ per minute	+	out-of-state minutes	·	5¢ per minute
Equation	B	=	20	+	m	·	0.06	+	$(300 - m)$	·	0.05

$= 20 + 0.06m + 300(0.05) - m(0.05)$ Distributive Property

$= 20 + 0.06m + 15 - 0.05m$ Simplify.

$= 35 + 0.01m$ Simplify.

An expression for Greg's phone bill for January is $35 + 0.01m$, where m is the number of minutes of in-state calls.

b. Evaluate the expression to find the cost if Greg had 37 minutes of in-state calls in January.

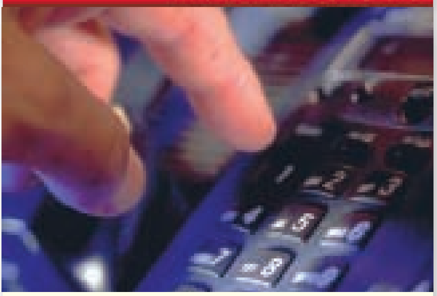
$35 + 0.01m = 35 + 0.01(37)$ $m = 37$

$= 35 + 0.37$ Multiply.

$= \$35.37$ Add.

Greg's bill was \$35.37.

More About. . .



Phone Service

About 98% of long-distance companies service their calls using the network of one of three companies. Since the quality of phone service is basically the same, a company's rates are the primary factor in choosing a long-distance provider.

Source: Chamberland Enterprises

SOLVE EQUATIONS WITH POLYNOMIAL EXPRESSIONS Many equations contain polynomials that must be added, subtracted, or multiplied before the equation can be solved.

Example 4 Polynomials on Both Sides

Solve $y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$.

$y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$ Original equation

$y^2 - 12y + y^2 + 2y + 25 = 2y^2 + 10y - 15$ Distributive Property

$2y^2 - 10y + 25 = 2y^2 + 10y - 15$ Combine like terms.

$-10y + 25 = 10y - 15$ Subtract $2y^2$ from each side.

$-20y + 25 = -15$ Subtract $10y$ from each side.

$-20y = -40$ Subtract 25 from each side.

$y = 2$ Divide each side by -20.

The solution is 2.

CHECK $y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$ Original equation

$2(2 - 12) + 2(2 + 2) + 25 \stackrel{?}{=} 2(2)(2 + 5) - 15$ $y = 2$

$2(-10) + 2(4) + 25 \stackrel{?}{=} 4(7) - 15$ Simplify.

$-20 + 8 + 25 \stackrel{?}{=} 28 - 15$ Multiply.

$13 = 13$ ✓ Add and subtract.

Check for Understanding

Concept Check

1. **State** the property used in each step to multiply $2x(4x^2 + 3x - 5)$.

$$2x(4x^2 + 3x - 5) = 2x(4x^2) + 2x(3x) - 2x(5) \quad \underline{\ ?\ }$$
$$= 8x^{1+2} + 6x^{1+1} - 10x \quad \underline{\ ?\ }$$
$$= 8x^3 + 6x^2 - 10x \quad \text{Simplify.}$$

2. **Compare and contrast** the procedure used to multiply a trinomial by a binomial using the vertical method with the procedure used to multiply a three-digit number by a two-digit number.
3. **OPEN ENDED** Write a monomial and a trinomial involving a single variable. Then find their product.

Guided Practice

Find each product.

4. $-3y(5y + 2)$
5. $9b^2(2b^3 - 3b^2 + b - 8)$
6. $2x(4a^4 - 3ax + 6x^2)$
7. $-4xy(5x^2 - 12xy + 7y^2)$

Simplify.

8. $t(5t - 9) - 2t$
9. $5n(4n^3 + 6n^2 - 2n + 3) - 4(n^2 + 7n)$

Solve each equation.

10. $-2(w + 1) + w = 7 - 4w$
11. $x(x + 2) - 3x = x(x - 4) + 5$

Application

SAVINGS **For Exercises 12–14, use the following information.**
Kenzie's grandmother left her $10,000 for college. Kenzie puts some of the money into a savings account earning 4% per year, and with the rest, she buys a certificate of deposit (CD) earning 7% per year.

12. If Kenzie puts x dollars into the savings account, write an expression to represent the amount of the CD.
13. Write an equation for the total amount of money T Kenzie will have saved for college after one year.
14. If Kenzie puts $3000 in savings, how much money will she have after one year?

Practice and Apply

Homework Help

For Exercises	See Examples
15–28	1
29–38	2
39–48	4
49–54, 58–62	3

Extra Practice
See page 838.

Find each product.

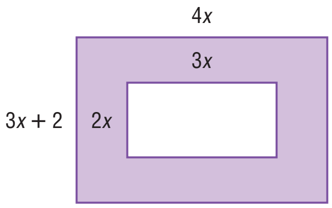
15. $r(5r + r^2)$
16. $w(2w^3 - 9w^2)$
17. $-4x(8 + 3x)$
18. $5y(-2y^2 - 7y)$
19. $7ag(g^3 + 2ag)$
20. $-3np(n^2 - 2p)$
21. $-2b^2(3b^2 - 4b + 9)$
22. $6x^3(5 + 3x - 11x^2)$
23. $8x^2y(5x + 2y^2 - 3)$
24. $-cd^2(3d + 2c^2d - 4c)$
25. $-\frac{3}{4}hk^2(20k^2 + 5h - 8)$
26. $\frac{2}{3}a^2b(6a^3 - 4ab + 9b^2)$
27. $-5a^3b(2b + 5ab - b^2 + a^3)$
28. $4p^2q^2(2p^2 - q^2 + 9p^3 + 3q)$

Simplify.

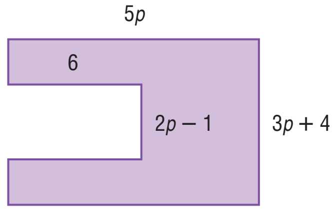
29. $d(-2d + 4) + 15d$
30. $-x(4x^2 - 2x) - 5x^3$
31. $3w(6w - 4) + 2(w^2 - 3w + 5)$
32. $5n(2n^3 + n^2 + 8) + n(4 - n)$
33. $10(4m^3 - 3m + 2) - 2m(-3m^2 - 7m + 1)$
34. $4y(y^2 - 8y + 6) - 3(2y^3 - 5y^2 + 2)$
35. $-3c^2(2c + 7) + 4c(3c^2 - c + 5) + 2(c^2 - 4)$
36. $4x^2(x + 2) + 3x(5x^2 + 2x - 6) - 5(3x^2 - 4x)$

GEOMETRY **Find the area of each shaded region in simplest form.**

37.



38.



Solve each equation.

39. $2(4x - 7) = 5(-2x - 9) - 5$

40. $2(5a - 12) = -6(2a - 3) + 2$

41. $4(3p + 9) - 5 = -3(12p - 5)$

42. $7(8w - 3) + 13 = 2(6w + 7)$

43. $d(d - 1) + 4d = d(d - 8)$

44. $c(c + 3) - c(c - 4) = 9c - 16$

45. $y(y + 12) - 8y = 14 + y(y - 4)$

46. $k(k - 7) + 10 = 2k + k(k + 6)$

47. $2n(n + 4) + 18 = n(n + 5) + n(n - 2) - 7$

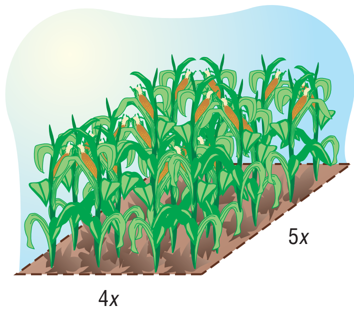
48. $3g(g - 4) - 2g(g - 7) = g(g + 6) - 28$

SAVINGS **For Exercises 49 and 50, use the following information.**
Marta has $6000 to invest. She puts x dollars of this money into a savings account that earns 3% per year, and with the rest, she buys a certificate of deposit that earns 6% per year.

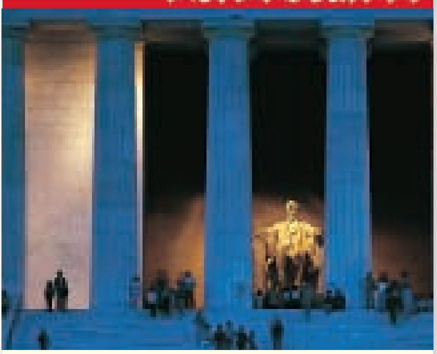
49. Write an equation for the total amount of money T Marta will have after one year.

50. Suppose at the end of one year, Marta has a total of $6315. How much money did Marta invest in each account?

51. **GARDENING** A gardener plants corn in a garden with a length-to-width ratio of 5:4. Next year, he plans to increase the garden's area by increasing its length by 12 feet. Write an expression for this new area.



More About. . .



Class Trip

Inside the Lincoln Memorial is a 19-foot marble statue of the United States' 16th president. The statue is flanked on either side by the inscriptions of Lincoln's Second Inaugural Address and Gettysburg Address.

Source: www.washington.org

52. **CLASS TRIP** Mr. Smith's American History class will take taxis from their hotel in Washington, D.C., to the Lincoln Memorial. The fare is $2.75 for the first mile and $1.25 for each additional mile. If the distance is m miles and t taxis are needed, write an expression for the cost to transport the group.

NUMBER THEORY **For Exercises 53 and 54, let x be an odd integer.**

53. Write an expression for the next odd integer.

54. Find the product of x and the next odd integer.

CRITICAL THINKING **For Exercises 55–57, use the following information.**
An even number can be represented by $2x$, where x is any integer.

55. Show that the product of two even integers is always even.

56. Write a representation for an odd integer.

57. Show that the product of an even and an odd integer is always even.

VOLUNTEERING **For Exercises 58 and 59, use the following information.**
Laura is making baskets of apples and oranges for homeless shelters. She wants to place a total of 10 pieces of fruit in each basket. Apples cost 25¢ each, and oranges cost 20¢ each.

58. If a represents the number of apples Laura uses, write a polynomial model in simplest form for the total amount of money T Laura will spend on the fruit for each basket.

59. If Laura uses 4 apples in each basket, find the total cost for fruit.

More About. . .

Volunteering

Approximately one third of young people in grades 7–12 suggested that "working for the good of my community and country" and "helping others or volunteering" were important future goals.

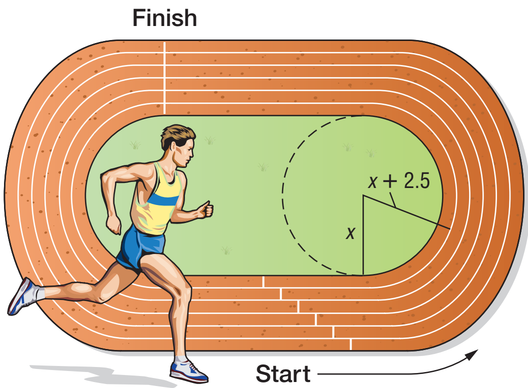
Source: Primedia/Roper National Youth Opinion Survey

SALES **For Exercises 60 and 61, use the following information.**
A store advertises that all sports equipment is 30% off the retail price. In addition, the store asks customers to select and pop a balloon to receive a coupon for an additional n percent off the already marked down price of one of their purchases.

60. Write an expression for the cost of a pair of inline skates with retail price p after receiving both discounts.

61. Use this expression to calculate the cost, not including sales tax, of a $200 pair of inline skates for an additional 10 percent off.

62. **SPORTS** You may have noticed that when runners race around a curved track, their starting points are staggered. This is so each contestant runs the same distance to the finish line.



If the radius of the inside lane is x and each lane is 2.5 feet wide, how far apart should the officials start the runners in the two inside lanes? (*Hint*: Circumference of a circle: $C = 2\pi r$, where r is the radius of the circle)

63. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How is finding the product of a monomial and a polynomial related to finding the area of a rectangle?

Include the following in your answer:
- the product of $2x$ and $x + 3$ derived algebraically, and
- a representation of another product of a monomial and a polynomial using algebra tiles and multiplication.

Standardized Test Practice

64. Simplify $[(3x^2 - 2x + 4) - (x^2 + 5x - 2)](x + 2)$.

Ⓐ $2x^3 + 7x^2 + 8x + 4$ Ⓑ $2x^3 - 3x^2 - 8x + 12$
Ⓒ $4x^3 + 11x^2 + 8x + 4$ Ⓓ $-4x^3 - 11x^2 - 8x - 4$

65. A plumber charges $70 for the first thirty minutes of each house call plus $4 for each additional minute that she works. The plumber charges Ke-Min $122 for her time. What amount of time, in minutes, did the plumber work?

Ⓐ 43 Ⓑ 48 Ⓒ 58 Ⓓ 64

Maintain Your Skills

Mixed Review

Find each sum or difference. *(Lesson 8-5)*

66. $(4x^2 + 5x) + (-7x^2 + x)$

67. $(3y^2 + 5y - 6) - (7y^2 - 9)$

68. $(5b - 7ab + 8a) - (5ab - 4a)$

69. $(6p^3 + 3p^2 - 7) + (p^3 - 6p^2 - 2p)$

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*. *(Lesson 8-4)*

70. $4x^2 - 10ab + 6$

71. $4c + ab - c$

72. $\frac{7}{y} + y^2$

73. $\frac{n^2}{3}$

Define a variable, write an inequality, and solve each problem. Then check your solution. *(Lesson 6-3)*

74. Six increased by ten times a number is less than nine times the number.

75. Nine times a number increased by four is no less than seven decreased by thirteen times the number.

Write an equation of the line that passes through each pair of points. *(Lesson 5-4)*

76. $(-3, -8), (1, 4)$

77. $(-4, 5), (2, -7)$

78. $(3, -1), (-3, 2)$

79. EXPENSES Kristen spent one fifth of her money on gasoline to fill up her car. Then she spent half of what was left for a haircut. She bought lunch for $7. When she got home, she had $13 left. How much money did Kristen have originally? *(Lesson 3-4)*

For Exercises 80 and 81, use each set of data to make a stem-and-leaf plot. *(Lesson 2-5)*

80. 49 51 55 62 47 32 56 57 48 47 33 68 53 45 30

81. 21 18 34 30 20 15 14 10 22 21 18 43 44 20 18

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review **products of polynomials**, see Lesson 8-1.)*

82. $(a)(a)$

83. $2x(3x^2)$

84. $-3y^2(8y^2)$

85. $4y(3y) - 4y(6)$

86. $-5n(2n^2) - (-5n)(8n) + (-5n)(4)$

87. $3p^2(6p^2) - 3p^2(8p) + 3p^2(12)$

Practice Quiz 2 — Lessons 8-4 through 8-6

Find the degree of each polynomial. *(Lesson 8-4)*

1. $5x^4$

2. $-9n^3p^4$

3. $7a^2 - 2ab^2$

4. $-6 - 8x^2y^2 + 5y^3$

Arrange the terms of each polynomial so that the powers of *x* are in ascending order. *(Lesson 8-4)*

5. $4x^2 + 9x - 12 + 5x^3$

6. $2xy^4 + x^3y^5 + 5x^5y - 13x^2$

Find each sum or difference. *(Lesson 8-5)*

7. $(7n^2 - 4n + 10) + (3n^2 - 8)$

8. $(3g^3 - 5g) - (2g^3 + 5g^2 - 3g + 1)$

Find each product. *(Lesson 8-6)*

9. $5a^2(3a^3b - 2a^2b^2 + 6ab^3)$

10. $7x^2y(5x^2 - 3xy + y)$

Algebra Activity

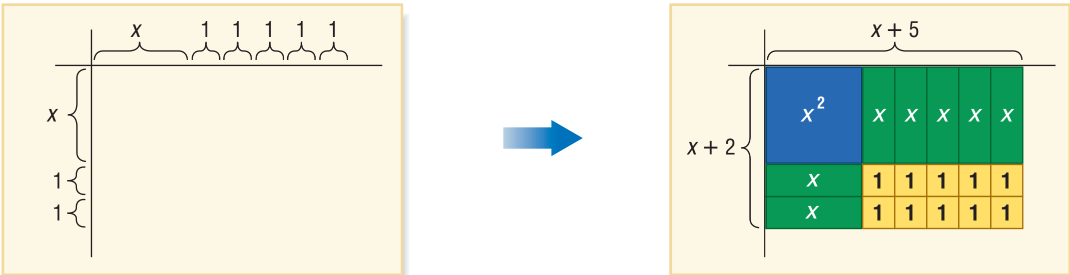
A Preview of Lesson 8-7

Multiplying Polynomials

You can use algebra tiles to find the product of two binomials.

Activity 1 Use algebra tiles to find $(x + 2)(x + 5)$.

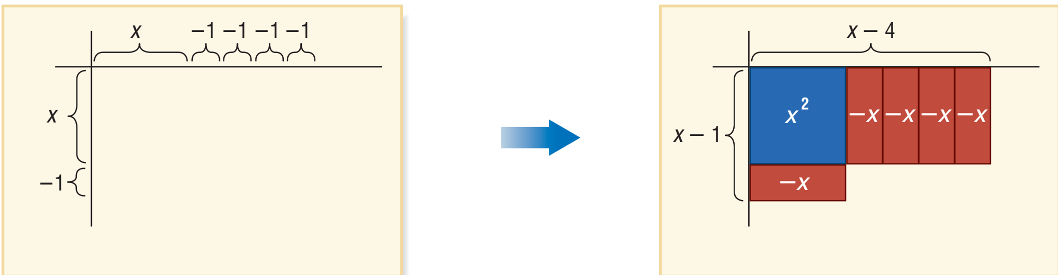
The rectangle will have a width of $x + 2$ and a length of $x + 5$. Use algebra tiles to mark off the dimensions on a product mat. Then complete the rectangle with algebra tiles.



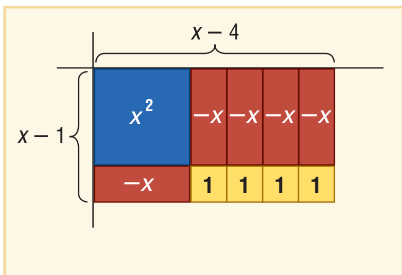
The rectangle consists of 1 blue x^2 tile, 7 green x tiles, and 10 yellow 1 tiles. The area of the rectangle is $x^2 + 7x + 10$. Therefore, $(x + 2)(x + 5) = x^2 + 7x + 10$.

Activity 2 Use algebra tiles to find $(x - 1)(x - 4)$.

Step 1 The rectangle will have a width of $x - 1$ and a length of $x - 4$. Use algebra tiles to mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.



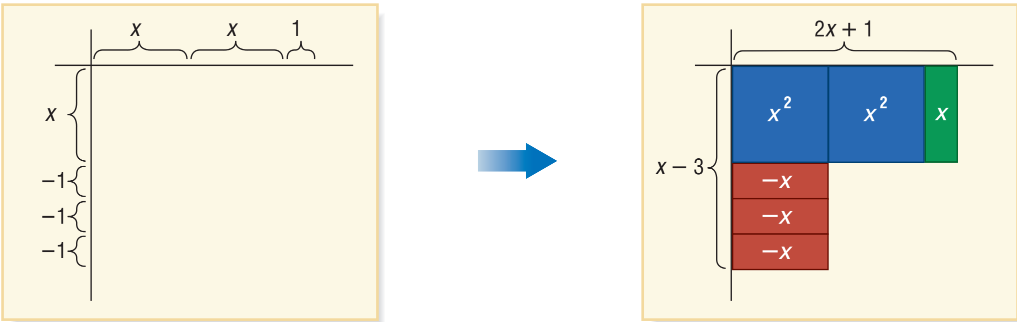
Step 2 Determine whether to use 4 yellow 1 tiles or 4 red -1 tiles to complete the rectangle. Remember that the numbers at the top and side give the dimensions of the tile needed. The area of each tile is the product of -1 and -1 or 1. This is represented by a yellow 1 tile. Fill in the space with 4 yellow 1 tiles to complete the rectangle.



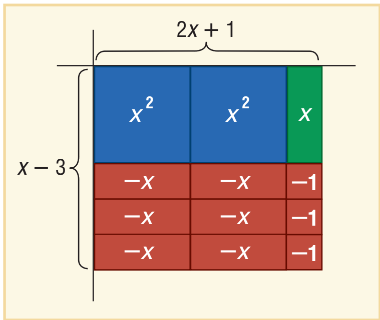
The rectangle consists of 1 blue x^2 tile, 5 red $-x$ tiles, and 4 yellow 1 tiles. The area of the rectangle is $x^2 - 5x + 4$. Therefore, $(x - 1)(x - 4) = x^2 - 5x + 4$.

Activity 3 Use algebra tiles to find $(x - 3)(2x + 1)$.

Step 1 The rectangle will have a width of $x - 3$ and a length of $2x + 1$. Mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.

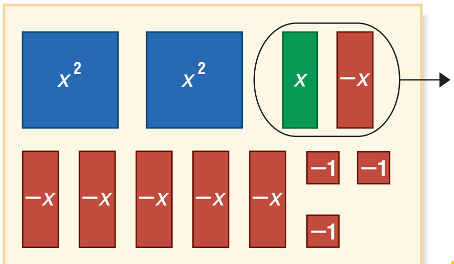


Step 2 Determine what color x tiles and what color 1 tiles to use to complete the rectangle. The area of each x tile is the product of x and -1. This is represented by a red $-x$ tile. The area of the 1 tile is represented by the product of 1 and -1 or -1. This is represented by a red -1 tile. Complete the rectangle with 3 red $-x$ tiles and 3 red -1 tiles.



Step 3 Rearrange the tiles to simplify the polynomial you have formed. Notice that a zero pair is formed by one positive and one negative x tile.

There are 2 blue x^2 tiles, 5 red $-x$ tiles, and 3 red -1 tiles left. In simplest form, $(x - 3)(2x + 1) = 2x^2 - 5x - 3$.



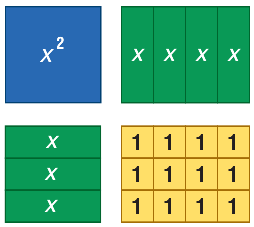
Model and Analyze

Use algebra tiles to find each product.

1. $(x + 2)(x + 3)$ **2.** $(x - 1)(x - 3)$ **3.** $(x + 1)(x - 2)$

4. $(x + 1)(2x + 1)$ **5.** $(x - 2)(2x - 3)$ **6.** $(x + 3)(2x - 4)$

7. You can also use the Distributive Property to find the product of two binomials. The figure at the right shows the model for $(x + 3)(x + 4)$ separated into four parts. Write a sentence or two explaining how this model shows the use of the Distributive Property.



8-7 Multiplying Polynomials

What You'll Learn

- Multiply two binomials by using the FOIL method.
- Multiply two polynomials by using the Distributive Property.

Vocabulary

- FOIL method

How is multiplying binomials similar to multiplying two-digit numbers?

To compute 24 × 36, we multiply each digit in 24 by each digit in 36, paying close attention to the place value of each digit.

Step 1 Multiply by the ones.	Step 2 Multiply by the tens.	Step 3 Add like place values.
$\begin{array}{r} 24 \\ \times\ 36 \\ \hline 144 \end{array}$	$\begin{array}{r} 24 \\ \times\ 36 \\ \hline 144 \\ 720 \\ \hline \end{array}$	$\begin{array}{r} 24 \\ \times\ 36 \\ \hline 144 \\ +\ 720 \\ \hline 864 \end{array}$
$6 \times 24 = 6(20 + 4)$ $= 120 + 24$ or 144	$30 \times 24 = 30(20 + 4)$ $= 600 + 120$ or 720	

You can multiply two binomials in a similar way.

MULTIPLY BINOMIALS To multiply two binomials, apply the Distributive Property twice as you do when multiplying two-digit numbers.

Study Tip

Look Back

To review the **Distributive Property**, see Lesson 1-7.

Example 1 The Distributive Property

Find $(x + 3)(x + 2)$.

Method 1 Vertical

Multiply by 2.	Multiply by x.	Add like terms.
$\begin{array}{r} x + 3 \\ (\times)\ x + 2 \\ \hline 2x + 6 \end{array}$	$\begin{array}{r} x + 3 \\ (\times)\ x + 2 \\ \hline 2x + 6 \\ x^2 + 3x \\ \hline \end{array}$	$\begin{array}{r} x + 3 \\ (\times)\ x + 2 \\ \hline 2x + 6 \\ x^2 + 3x \\ \hline x^2 + 5x + 6 \end{array}$
$2(x + 3) = 2x + 6$	$x(x + 3) = x^2 + 3x$	

Method 2 Horizontal

$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$ Distributive Property

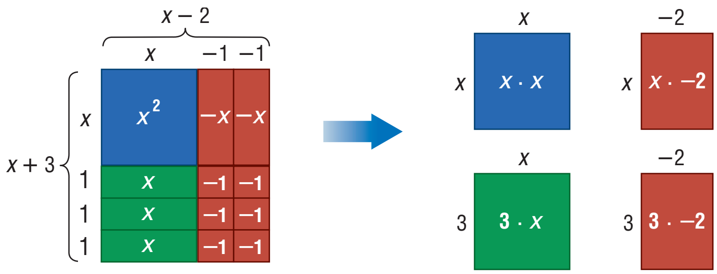
$= x(x) + x(2) + 3(x) + 3(2)$ Distributive Property

$= x^2 + 2x + 3x + 6$ Multiply.

$= x^2 + 5x + 6$ Combine like terms.

An alternative method for finding the product of two binomials can be shown using algebra tiles.

Consider the product of $x + 3$ and $x - 2$. The rectangle shown below has a length of $x + 3$ and a width of $x - 2$. Notice that this rectangle can be broken up into four smaller rectangles.



The product of $(x - 2)$ and $(x + 3)$ is the sum of these four areas.

$(x + 3)(x - 2) = (x \cdot x) + (x \cdot -2) + (3 \cdot x) + (3 \cdot -2)$ Sum of the four areas

$= x^2 + (-2x) + 3x + (-6)$ Multiply.

$= x^2 + x - 6$ Combine like terms.

This example illustrates a shortcut of the Distributive Property called the **FOIL method**. You can use the FOIL method to multiply two binomials.

Key Concept — *FOIL Method for Multiplying Binomials*

- **Words** To multiply two binomials, find the sum of the products of
  - F the *First* terms,
  - O the *Outer* terms,
  - I the *Inner* terms, and
  - L the *Last* terms.
- **Example**

	Product of First terms		Product of Outer terms		Product of Inner terms		Product of Last terms
F L / I O: $(x + 3)(x - 2) =$	$(x)(x)$	+	$(-2)(x)$	+	$(3)(x)$	+	$(3)(-2)$

$= x^2 - 2x + 3x - 6$

$= x^2 + x - 6$

Example 2 *FOIL Method*

Find each product.

a. $(x - 5)(x + 7)$

F L / I O

$(x - 5)(x + 7) = \overset{F}{(x)(x)} + \overset{O}{(x)(7)} + \overset{I}{(-5)(x)} + \overset{L}{(-5)(7)}$ FOIL method

$= x^2 + 7x - 5x - 35$ Multiply.

$= x^2 + 2x - 35$ Combine like terms.

b. $(2y + 3)(6y - 7)$

$(2y + 3)(6y - 7)$

$= \overset{F}{(2y)(6y)} + \overset{O}{(2y)(-7)} + \overset{I}{(3)(6y)} + \overset{L}{(3)(-7)}$ FOIL method

$= 12y^2 - 14y + 18y - 21$ Multiply.

$= 12y^2 + 4y - 21$ Combine like terms.

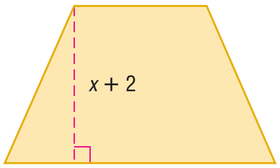
Study Tip

Checking Your Work

You can check your products in Examples 2a and 2b by reworking each problem using the Distributive Property.

Example 3 FOIL Method

GEOMETRY The area A of a trapezoid is one-half the height h times the sum of the bases, b_1 and b_2. Write an expression for the area of the trapezoid.



Identify the height and bases.

$h = x + 2$

$b_1 = 3x - 7$

$b_2 = 2x + 1$

Now write and apply the formula.

Area	equals	one-half		height	times	sum of bases,
A	$=$	$\frac{1}{2}$	$\cdot$	h	$\cdot$	$(b_1 + b_2)$

$A = \frac{1}{2}h(b_1 + b_2)$ Original formula

$= \frac{1}{2}(x + 2)[(3x - 7) + (2x + 1)]$ Substitution

$= \frac{1}{2}(x + 2)(5x - 6)$ Add polynomials in the brackets.

$= \frac{1}{2}[x(5x) + x(-6) + 2(5x) + 2(-6)]$ FOIL method

$= \frac{1}{2}(5x^2 - 6x + 10x - 12)$ Multiply.

$= \frac{1}{2}(5x^2 + 4x - 12)$ Combine like terms.

$= \frac{5}{2}x^2 + 2x - 6$ Distributive Property

The area of the trapezoid is $\frac{5}{2}x^2 + 2x - 6$ square units.

MULTIPLY POLYNOMIALS The Distributive Property can be used to multiply any two polynomials.

Example 4 The Distributive Property

Find each product.

a. $(4x + 9)(2x^2 - 5x + 3)$

$(4x + 9)(2x^2 - 5x + 3)$

$= 4x(2x^2 - 5x + 3) + 9(2x^2 - 5x + 3)$ Distributive Property

$= 8x^3 - 20x^2 + 12x + 18x^2 - 45x + 27$ Distributive Property

$= 8x^3 - 2x^2 - 33x + 27$ Combine like terms.

b. $(y^2 - 2y + 5)(6y^2 - 3y + 1)$

$(y^2 - 2y + 5)(6y^2 - 3y + 1)$

$= y^2(6y^2 - 3y + 1) - 2y(6y^2 - 3y + 1) + 5(6y^2 - 3y + 1)$ Distributive Property

$= 6y^4 - 3y^3 + y^2 - 12y^3 + 6y^2 - 2y + 30y^2 - 15y + 5$ Distributive Property

$= 6y^4 - 15y^3 + 37y^2 - 17y + 5$ Combine like terms.

Study Tip

Common Misconception

A common mistake when multiplying polynomials horizontally is to combine terms that are not alike. For this reason, you may prefer to multiply polynomials in column form, aligning like terms.

Check for Understanding

Concept Check

1. **Draw a diagram** to show how you would use algebra tiles to find the product of $2x - 1$ and $x + 3$.
2. **Show** how to find $(3x + 4)(2x - 5)$ using each method.
   a. Distributive Property
   b. FOIL method
   c. vertical or column method
   d. algebra tiles
3. **OPEN ENDED** State which method of multiplying binomials you prefer and why.

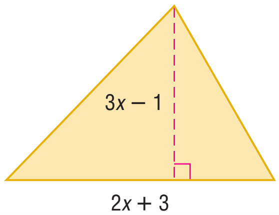
Guided Practice

Find each product.

4. $(y + 4)(y + 3)$
5. $(x - 2)(x + 6)$
6. $(a - 8)(a + 5)$
7. $(4h + 5)(h + 7)$
8. $(9p - 1)(3p - 2)$
9. $(2g + 7)(5g - 8)$
10. $(3b - 2c)(6b + 5c)$
11. $(3k - 5)(2k^2 + 4k - 3)$

Application

12. **GEOMETRY** The area A of a triangle is half the product of the base b times the height h. Write a polynomial expression that represents the area of the triangle at the right.



Practice and Apply

Homework Help

For Exercises	See Examples
13–38	1, 2, 4
39–42	3

Extra Practice

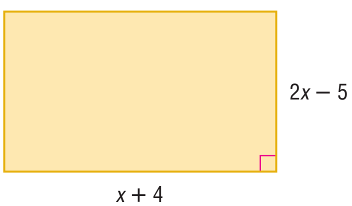
See page 839.

Find each product.

13. $(b + 8)(b + 2)$
14. $(n + 6)(n + 7)$
15. $(x - 4)(x - 9)$
16. $(a - 3)(a - 5)$
17. $(y + 4)(y - 8)$
18. $(p + 2)(p - 10)$
19. $(2w - 5)(w + 7)$
20. $(k + 12)(3k - 2)$
21. $(8d + 3)(5d + 2)$
22. $(4g + 3)(9g + 6)$
23. $(7x - 4)(5x - 1)$
24. $(6a - 5)(3a - 8)$
25. $(2n + 3)(2n + 3)$
26. $(5m - 6)(5m - 6)$
27. $(10r - 4)(10r + 4)$
28. $(7t + 5)(7t - 5)$
29. $(8x + 2y)(5x - 4y)$
30. $(11a - 6b)(2a + 3b)$
31. $(p + 4)(p^2 + 2p - 7)$
32. $(a - 3)(a^2 - 8a + 5)$
33. $(2x - 5)(3x^2 - 4x + 1)$
34. $(3k + 4)(7k^2 + 2k - 9)$
35. $(n^2 - 3n + 2)(n^2 + 5n - 4)$
36. $(y^2 + 7y - 1)(y^2 - 6y + 5)$
37. $(4a^2 + 3a - 7)(2a^2 - a + 8)$
38. $(6x^2 - 5x + 2)(3x^2 + 2x + 4)$

GEOMETRY **Write an expression to represent the area of each figure.**

39.



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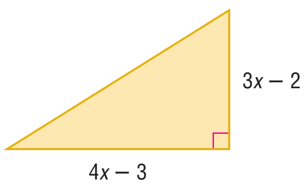

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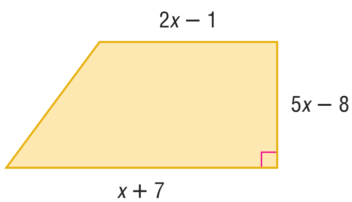
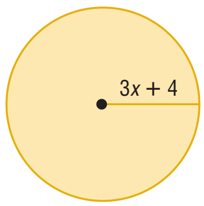
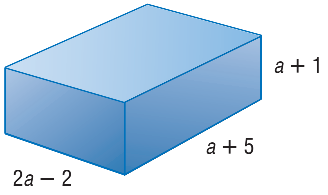


42.

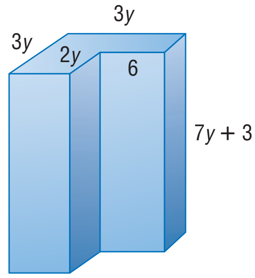


GEOMETRY **The volume V of a prism equals the area of the base B times the height h. Write an expression to represent the volume of each prism.**

43.



44.



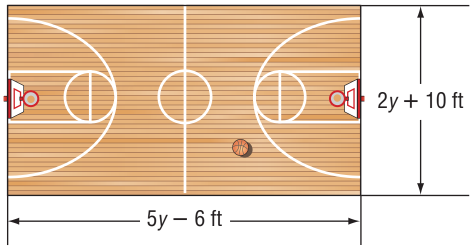
NUMBER THEORY **For Exercises 45–47, consider three consecutive integers. Let the least of these integers be a.**

45. Write a polynomial representing the product of these three integers.

46. Choose an integer for a. Find their product.

47. Evaluate the polynomial in Exercise 45 for the value of a you chose in Exercise 46. Describe the result.

48. **BASKETBALL** The dimensions of a professional basketball court are represented by a width of $2y + 10$ feet and a length of $5y - 6$ feet. Find an expression for the area of the court.



More About. . .

Basketball

More than 200 million people a year pay to see basketball games. That is more admissions than for any other American sport.

Source: *Compton's Encyclopedia*

OFFICE SPACE **For Exercises 49–51, use the following information.**
Latanya's modular office is square. Her office in the company's new building will be 2 feet shorter in one direction and 4 feet longer in the other.

49. Write expressions for the dimensions of Latanya's new office.

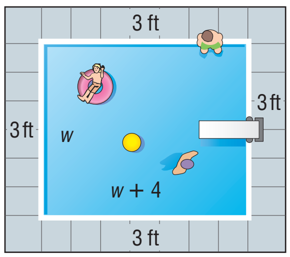
50. Write a polynomial expression for the area of her new office.

51. Suppose her office is presently 9 feet by 9 feet. Will her new office be bigger or smaller than her old office and by how much?

52. **MENTAL MATH** One way to mentally multiply 25 and 18 is to find $(20 + 5)(20 - 2)$. Show how the FOIL method can be used to find each product.

a. 35(19) **b.** 67(102) **c.** $8\frac{1}{2} \cdot 6\frac{3}{4}$ **d.** $12\frac{3}{5} \cdot 10\frac{2}{3}$

53. **POOL CONSTRUCTION** A homeowner is installing a swimming pool in his backyard. He wants its length to be 4 feet longer than its width. Then he wants to surround it with a concrete walkway 3 feet wide. If he can only afford 300 square feet of concrete for the walkway, what should the dimensions of the pool be?



54. **CRITICAL THINKING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

The product of a binomial and a trinomial is a polynomial with four terms.

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is multiplying binomials similar to multiplying two-digit numbers?

Include the following in your answer:

- a demonstration of a horizontal method for multiplying 24×36, and
- an explanation of the meaning of "like terms" in the context of vertical two-digit multiplication.

Standardized Test Practice

56. $(x + 2)(x - 4) - (x + 4)(x - 2) =$

Ⓐ 0 Ⓑ $2x^2 + 4x - 16$ Ⓒ $-4x$ Ⓓ $4x$

57. The expression $(x - y)(x^2 + xy + y^2)$ is equivalent to which of the following?

Ⓐ $x^2 - y^2$ Ⓑ $x^3 - y^3$ Ⓒ $x^3 - xy^2$ Ⓓ $x^3 - x^2y + y^2$

Maintain Your Skills

Mixed Review

Find each product. *(Lesson 8-6)*

58. $3d(4d^2 - 8d - 15)$ **59.** $-4y(7y^2 - 4y + 3)$ **60.** $2m^2(5m^2 - 7m + 8)$

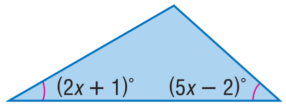
Simplify. *(Lesson 8-6)*

61. $3x(2x - 4) + 6(5x^2 + 2x - 7)$ **62.** $4a(5a^2 + 2a - 7) - 3(2a^2 - 6a - 9)$

GEOMETRY **For Exercises 63 and 64, use the following information.**
The sum of the degree measures of the angles of a triangle is 180. *(Lesson 8-5)*

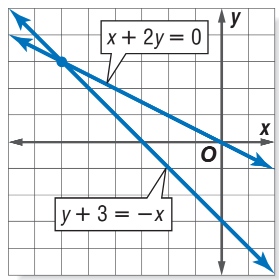
63. Write an expression to represent the measure of the third angle of the triangle.

64. If $x = 15$, find the measures of the three angles of the triangle.



65. Use the graph at the right to determine whether the system below has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. *(Lesson 7-1)*

$x + 2y = 0$
$y + 3 = -x$



If $f(x) = 2x - 5$ and $g(x) = x^2 + 3x$, find each value. *(Lesson 4-6)*

66. $f(-4)$ **67.** $g(-2) + 7$ **68.** $f(a + 3)$

Solve each equation or formula for the variable specified. *(Lesson 3-8)*

69. $a = \frac{v}{t}$ for t **70.** $ax - by = 2cz$ for y **71.** $4x + 3y = 7$ for y

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.**
(To review ***Power of a Power*** *and* ***Power of a Product*** *Properties, see Lesson 8-1.)*

72. $(6a)^2$ **73.** $(7x)^2$ **74.** $(9b)^2$

75. $(4y^2)^2$ **76.** $(2v^3)^2$ **77.** $(3g^4)^2$

CONTENTS

8-8 Special Products

What You'll Learn

- Find squares of sums and differences.
- Find the product of a sum and a difference.

Vocabulary

- difference of squares

When is the product of two binomials also a binomial?

In the previous lesson, you learned how to multiply two binomials using the FOIL method. You may have noticed that the *Outer* and *Inner* terms often combine to produce a trinomial product.

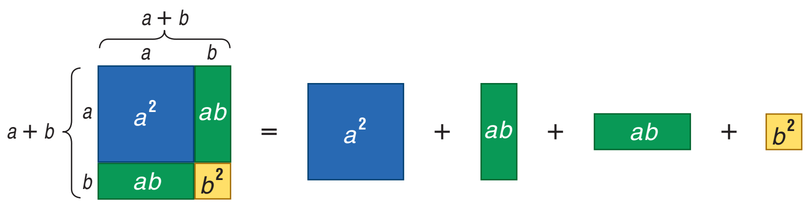
$$\begin{aligned}(x+5)(x-3) &= \overset{F}{x^2} \overset{O}{- 3x} \overset{I}{+ 5x} \overset{L}{- 15} \\ &= x^2 + 2x - 15 \qquad \text{Combine like terms.}\end{aligned}$$

This is not always the case, however. Examine the product below.

$$\begin{aligned}(x+3)(x-3) &= \overset{F}{x^2} \overset{O}{- 3x} \overset{I}{+ 3x} \overset{L}{- 9} \\ &= x^2 + 0x - 9 \qquad \text{Combine like terms.} \\ &= x^2 - 9 \qquad \text{Simplify.}\end{aligned}$$

Notice that the product of $x + 3$ and $x - 3$ is a binomial.

SQUARES OF SUMS AND DIFFERENCES While you can always use the FOIL method to find the product of two binomials, some pairs of binomials have products that follow a specific pattern. One such pattern is the *square of a sum*, $(a + b)^2$ or $(a + b)(a + b)$. You can use the diagram below to derive the pattern for this special product.



$$\begin{aligned}(a+b)^2 &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

Key Concept — Square of a Sum

- **Words** The square of $a + b$ is the square of a plus twice the product of a and b plus the square of b.
- **Symbols** $(a + b)^2 = (a + b)(a + b)$
  $= a^2 + 2ab + b^2$
- **Example** $(x + 7)^2 = x^2 + 2(x)(7) + 7^2$
  $= x^2 + 14x + 49$

Example 1 Square of a Sum

Find each product.

a. $(4y + 5)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ — Square of a Sum

$(4y + 5)^2 = (4y)^2 + 2(4y)(5) + 5^2$ — $a = 4y$ and $b = 5$

$= 16y^2 + 40y + 25$ — Simplify.

CHECK Check your work by using the FOIL method.

$(4y + 5)^2 = (4y + 5)(4y + 5)$

$= \overset{F}{(4y)(4y)} + \overset{O}{(4y)(5)} + \overset{I}{5(4y)} + \overset{L}{5(5)}$

$= 16y^2 + 20y + 20y + 25$

$= 16y^2 + 40y + 25$ ✓

b. $(8c + 3d)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ — Square of a Sum

$(8c + 3d)^2 = (8c)^2 + 2(8c)(3d) + (3d)^2$ — $a = 8c$ and $b = 3d$

$= 64c^2 + 48cd + 9d^2$ — Simplify.

Study Tip

$(a + b)^2$

In the pattern for $(a + b)^2$, a and b can be numbers, variables, or expressions with numbers and variables.

To find the pattern for the *square of a difference*, $(a - b)^2$, write $a - b$ as $a + (-b)$ and square it using the square of a sum pattern.

$(a - b)^2 = [a + (-b)]^2$

$= a^2 + 2(a)(-b) + (-b)^2$ — Square of a Sum

$= a^2 - 2ab + b^2$ — Simplify. Note that $(-b)^2 = (-b)(-b)$ or b^2.

The square of a difference can be found by using the following pattern.

Key Concept — Square of a Difference

- **Words** The square of $a - b$ is the square of a minus twice the product of a and b plus the square of b.
- **Symbols** $(a - b)^2 = (a - b)(a - b)$
  $= a^2 - 2ab + b^2$
- **Example** $(x - 4)^2 = x^2 - 2(x)(4) + 4^2$
  $= x^2 - 8x + 16$

Example 2 Square of a Difference

Find each product.

a. $(6p - 1)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ — Square of a Difference

$(6p - 1)^2 = (6p)^2 - 2(6p)(1) + 1^2$ — $a = 6p$ and $b = 1$

$= 36p^2 - 12p + 1$ — Simplify.

b. $(5m^3 - 2n)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ — Square of a Difference

$(5m^3 - 2n)^2 = (5m^3)^2 - 2(5m^3)(2n) + (2n)^2$ — $a = 5m^3$ and $b = 2n$

$= 25m^6 - 20m^3n + 4n^2$ — Simplify.

www.algebra1.com/extra_examples

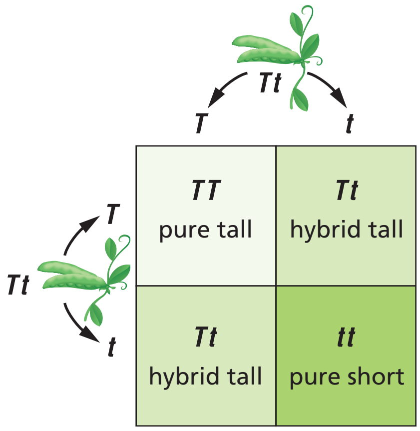
Geneticist

Laboratory geneticists work in medicine to find cures for disease, in agriculture to breed new crops and livestock, and in police work to identify criminals.

Online Research For information about a career as a geneticist, visit: www.algebra1.com/careers

Example 3 Apply the Sum of a Square

GENETICS **The Punnett square shows the possible gene combinations of a cross between two pea plants. Each plant passes along one *dominant* gene *T* for tallness and one *recessive* gene *t* for shortness.**



Show how combinations can be modeled by the square of a binomial. Then determine what percent of the offspring will be pure tall, hybrid tall, and pure short.

Each parent has half the genes necessary for tallness and half the genes necessary for shortness. The makeup of each parent can be modeled by $0.5T + 0.5t$. Their offspring can be modeled by the product of $0.5T + 0.5t$ and $0.5T + 0.5t$ or $(0.5T + 0.5t)^2$.

If we expand this product, we can determine the possible heights of the offspring.

$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{Square of a Sum}$$

$$(0.5T + 0.5t)^2 = (0.5T)^2 + 2(0.5T)(0.5t) + (0.5t)^2 \quad a = 0.5T \text{ and } b = 0.5t$$

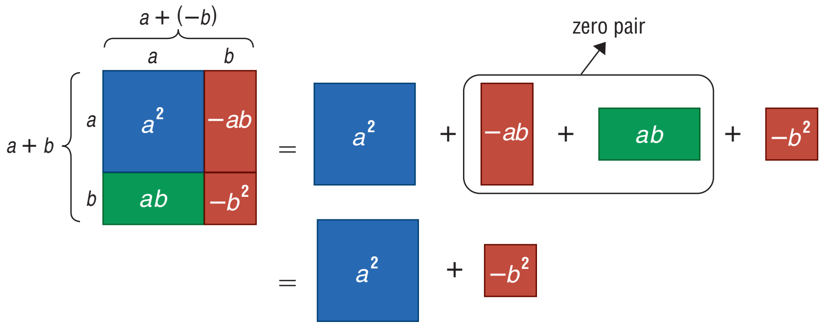
$$= 0.25T^2 + 0.5Tt + 0.25t^2 \quad \text{Simplify.}$$

$$= 0.25TT + 0.5Tt + 0.25tt \quad T^2 = TT \text{ and } t^2 = tt$$

Thus, 25% of the offspring are *TT* or pure tall, 50% are *Tt* or hybrid tall, and 25% are *tt* or pure short.

PRODUCT OF A SUM AND A DIFFERENCE

You can use the diagram below to find the pattern for the product of a sum and a difference of the *same two terms*, $(a + b)(a - b)$. Recall that $a - b$ can be rewritten as $a + (-b)$.



$$= a^2 + (-b^2)$$

$$= a^2 - b^2$$

The resulting product, $a^2 - b^2$, has a special name. It is called a **difference of squares**. Notice that this product has no middle term.

Key Concept — Product of a Sum and a Difference

- **Words** The product of $a + b$ and $a - b$ is the square of a minus the square of b.
- **Symbols** $(a + b)(a - b) = (a - b)(a + b)$
  $= a^2 - b^2$
- **Example** $(x + 9)(x - 9) = x^2 - 9^2$
  $= x^2 - 81$

Example 4 Product of a Sum and a Difference

Find each product.

a. $(3n + 2)(3n - 2)$

$(a + b)(a - b) = a^2 - b^2$ — Product of a Sum and a Difference

$(3n + 2)(3n - 2) = (3n)^2 - 2^2$ — $a = 3n$ and $b = 2$

$= 9n^2 - 4$ — Simplify.

b. $(11v - 8w^2)(11v + 8w^2)$

$(a - b)(a + b) = a^2 - b^2$ — Product of a Sum and a Difference

$(11v - 8w^2)(11v + 8w^2) = (11v)^2 - (8w^2)^2$ — $a = 11v$ and $b = 8w^2$

$= 121v^2 - 64w^4$ — Simplify.

The following list summarizes the special products you have studied.

Key Concept — Special Products

- **Square of a Sum** $(a + b)^2 = a^2 + 2ab + b^2$
- **Square of a Difference** $(a - b)^2 = a^2 - 2ab + b^2$
- **Product of a Sum and a Difference** $(a - b)(a + b) = a^2 - b^2$

Check for Understanding

Concept Check

1. **Compare and contrast** the pattern for the square of a sum with the pattern for the square of a difference.
2. **Explain** how the square of a difference and the difference of squares differ.
3. **Draw a diagram** to show how you would use algebra tiles to model the product of $x - 3$ and $x - 3$, or $(x - 3)^2$.
4. **OPEN ENDED** Write two binomials whose product is a difference of squares.

Guided Practice

Find each product.

5. $(a + 6)^2$
6. $(4n - 3)(4n - 3)$
7. $(8x - 5)(8x + 5)$
8. $(3a + 7b)(3a - 7b)$
9. $(x^2 - 6y)^2$
10. $(9 - p)^2$

Application

GENETICS For Exercises 11 and 12, use the following information.
In hamsters, golden coloring G is dominant over cinnamon coloring g. Suppose a purebred cinnamon male is mated with a purebred golden female.

11. Write an expression for the genetic makeup of the hamster pups.
12. What is the probability that the pups will have cinnamon coloring? Explain your reasoning.


Golden


Cinnamon

Practice and Apply

Homework Help

For Exercises	See Examples
13–38	1, 2, 4
39, 40	3

Extra Practice

See page 839.

Find each product.

13. $(y + 4)^2$
14. $(k + 8)(k + 8)$
15. $(a - 5)(a - 5)$
16. $(n - 12)^2$
17. $(b + 7)(b - 7)$
18. $(c - 2)(c + 2)$
19. $(2g + 5)^2$
20. $(9x + 3)^2$
21. $(7 - 4y)^2$
22. $(4 - 6h)^2$
23. $(11r + 8)(11r - 8)$
24. $(12p - 3)(12p + 3)$
25. $(a + 5b)^2$
26. $(m + 7n)^2$
27. $(2x - 9y)^2$
28. $(3n - 10p)^2$
29. $(5w + 14)(5w - 14)$
30. $(4d - 13)(4d + 13)$
31. $(x^3 + 4y)^2$
32. $(3a^2 - b^2)^2$
33. $(8a^2 - 9b^3)(8a^2 + 9b^3)$
34. $(5x^4 - y)(5x^4 + y)$
35. $\left(\frac{2}{3}x - 6\right)^2$
36. $\left(\frac{4}{5}x + 10\right)^2$
37. $(2n + 1)(2n - 1)(n + 5)$
38. $(p + 3)(p - 4)(p - 3)(p + 4)$

GENETICS **For Exercises 39 and 40, use the following information.**
Pam has brown eyes and Bob has blue eyes. Brown genes B are dominant over blue genes b. A person with genes BB or Bb has brown eyes. Someone with genes bb has blue eyes. Suppose Pam's genes for eye color are Bb.

39. Write an expression for the possible eye coloring of Pam and Bob's children.
40. What is the probability that a child of Pam and Bob would have blue eyes?

MAGIC TRICK **For Exercises 41–44, use the following information.**
Julie says that she can perform a magic trick with numbers. She asks you to pick a whole number, any whole number. Square that number. Then, add twice your original number. Next add 1. Take the square root of the result. Finally, subtract your original number. Then Julie exclaims with authority, "Your answer is 1!"

41. Pick a whole number and follow Julie's directions. Is your result 1?
42. Let a represent the whole number you chose. Then, find a polynomial representation for the first three steps of Julie's directions.
43. The polynomial you wrote in Exercise 42 is the square of what binomial sum?
44. Take the square root of the perfect square you wrote in Exercise 43, then subtract a, your original number. What is the result?

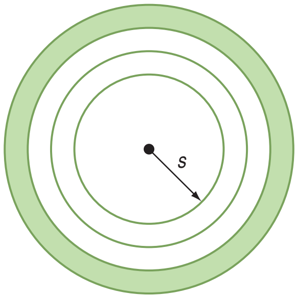
More About. . .

Architecture

The historical Gwennap Pit, an outdoor amphitheater in southern England, consists of a circular stage surrounded by circular levels used for seating. Each seating level is about 1 meter wide.

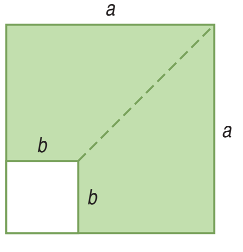
Source: *Christian Guide to Britain*

ARCHITECTURE **For Exercises 45 and 46, use the following information.**
A diagram of a portion of the Gwennap Pit is shown at the right. Suppose the radius of the stage is s meters.



45. Use the information at the left to find binomial representations for the radii of the second and third seating levels.
46. Find the area of the shaded region representing the third seating level.

47. **GEOMETRY** The area of the shaded region models the difference of two squares, $a^2 - b^2$. Show that the area of the shaded region is also equal to $(a - b)(a + b)$. (*Hint*: Divide the shaded region into two trapezoids as shown.)



48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

When is the product of two binomials also a binomial?

Include the following in your answer:

- an example of two binomials whose product is a binomial, and
- an example of two binomials whose product is not a binomial.

Standardized Test Practice

49. If $a^2 + b^2 = 40$ and $ab = 12$, find the value of $(a - b)^2$.

Ⓐ 1 Ⓑ 121 Ⓒ 16 Ⓓ 28

50. If $x - y = 10$ and $x + y = 20$, find the value of $x^2 - y^2$.

Ⓐ 400 Ⓑ 200 Ⓒ 100 Ⓓ 30

Extending the Lesson

51. Does a pattern exist for the cube of a sum, $(a + b)^3$?

a. Investigate this question by finding the product of $(a + b)(a + b)(a + b)$.

b. Use the pattern you discovered in part **a** to find $(x + 2)^3$.

c. Draw a diagram of a geometric model for the cube of a sum.

Maintain Your Skills

Mixed Review

Find each product. *(Lesson 8-7)*

52. $(x + 2)(x + 7)$ 53. $(c - 9)(c + 3)$ 54. $(4y - 1)(5y - 6)$

55. $(3n - 5)(8n + 5)$ 56. $(x - 2)(3x^2 - 5x + 4)$ 57. $(2k + 5)(2k^2 - 8k + 7)$

Solve. *(Lesson 8-6)*

58. $6(x + 2) + 4 = 5(3x - 4)$ 59. $-3(3a - 8) + 2a = 4(2a + 1)$

60. $p(p + 2) + 3p = p(p - 3)$ 61. $y(y - 4) + 2y = y(y + 12) - 7$

Use elimination to solve each system of equations. *(Lessons 7-3 and 7-4)*

62. $\frac{3}{4}x + \frac{1}{5}y = 5$, $\frac{3}{4}x - \frac{1}{5}y = -5$

63. $2x - y = 10$, $5x + 3y = 3$

64. $2x = 4 - 3y$, $3y - x = -11$

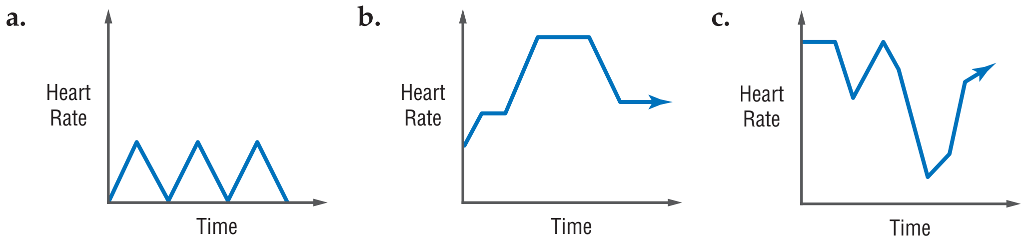
Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation. *(Lesson 5-6)*

65. $5x + 5y = 35, (-3, 2)$ 66. $2x - 5y = 3, (-2, 7)$ 67. $5x + y = 2, (0, 6)$

Find the *n*th term of each arithmetic sequence described. *(Lesson 4-7)*

68. $a_1 = 3, d = 4, n = 18$ 69. $-5, 1, 7, 13, \ldots$ for $n = 12$

70. **PHYSICAL FITNESS** Mitchell likes to exercise regularly. He likes to warm up by walking two miles. Then he runs five miles. Finally, he cools down by walking for another mile. Identify the graph that best represents Mitchell's heart rate as a function of time. *(Lesson 1-8)*



Chapter 8

Study Guide and Review

Vocabulary and Concept Check

binomial (p. 432)
constant (p. 410)
degree of a monomial (p. 433)
degree of a polynomial (p. 433)
difference of squares (p. 460)
FOIL method (p. 453)
monomial (p. 410)
negative exponent (p. 419)
polynomial (p. 432)
Power of a Power (p. 411)
Power of a Product (p. 412)
Power of a Quotient (p. 418)
Product of Powers (p. 411)
Quotient of Powers (p. 417)
scientific notation (p. 425)
trinomial (p. 432)
zero exponent (p. 419)

Choose a term from the vocabulary list that best matches each example.

1. $4^{-3} = \frac{1}{4^3}$

2. $(n^3)^5 = n^{15}$

3. $\frac{4x^2y}{8xy^3} = \frac{x}{2y^2}$

4. $4x^2$

5. $x^2 - 3x + 1$

6. $2^0 = 1$

7. $x^4 - 3x^3 + 2x^2 - 1$

8. $(x + 3)(x - 4) = x^2 - 4x + 3x - 12$

9. $x^2 + 2$

10. $(a^3b)(2ab^2) = 2a^4b^3$

Lesson-by-Lesson Review

8-1 Multiplying Monomials

See pages 410–415.

Concept Summary

Concept	Examples
• A monomial is a number, a variable, or a product of a number and one or more variables.	$6x^2, -5, \frac{2c}{3}$
• To multiply two powers that have the same base, add exponents.	$a^2 \cdot a^3 = a^5$
• To find the power of a power, multiply exponents.	$(a^2)^3 = a^6$
• The power of a product is the product of the powers.	$(ab^2)^3 = a^3b^6$

Examples

1 **Simplify $(2ab^2)(3a^2b^3)$.**

$(2ab^2)(3a^2b^3) = (2 \cdot 3)(a \cdot a^2)(b^2 \cdot b^3)$ Commutative Property
$= 6a^3b^5$ Product of Powers

2 **Simplify $(2x^2y^3)^3$.**

$(2x^2y^3)^3 = 2^3(x^2)^3(y^3)^3$ Power of a Product
$= 8x^6y^9$ Power of a Power

Exercises **Simplify.** *See Examples 2, 3, and 5 on pages 411 and 412.*

11. $y^3 \cdot y^3 \cdot y$

12. $(3ab)(-4a^2b^3)$

13. $(-4a^2x)(-5a^3x^4)$

14. $(4a^2b)^3$

15. $(-3xy)^2(4x)^3$

16. $(-2c^2d)^4(-3c^2)^3$

17. $-\frac{1}{2}(m^2n^4)^2$

18. $(5a^2)^3 + 7(a^6)$

19. $[(3^2)^2]^3$

www.algebra1.com/vocabulary_review

8-2 Dividing Monomials

See pages 417–423.

Concept Summary

- To divide two powers that have the same base, subtract the exponents.
- To find the power of a quotient, find the power of the numerator and the power of the denominator.
- Any nonzero number raised to the zero power is 1.
- For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

Examples

$\frac{a^5}{a^3} = a^2$

$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$

$(3a^3b^2)^0 = 1$

$a^{-3} = \frac{1}{a^3}$

Example **Simplify $\frac{2x^6y}{8x^2y^2}$. Assume that x and y are not equal to zero.**

$\frac{2x^6y}{8x^2y^2} = \left(\frac{2}{8}\right)\left(\frac{x^6}{x^2}\right)\left(\frac{y}{y^2}\right)$ Group the powers with the same base.

$= \left(\frac{1}{4}\right)(x^{6-2})(y^{1-2})$ Quotient of Powers

$= \frac{x^4}{4y}$ Simplify.

Exercises **Simplify. Assume that no denominator is equal to zero.**

See Examples 1–4 on pages 417–420.

20. $\frac{(3y)^0}{6a}$ **21.** $\left(\frac{3bc^2}{4d}\right)^3$ **22.** $x^{-2}y^0z^3$

23. $\frac{27b^{-2}}{14b^{-3}}$ **24.** $\frac{(3a^3bc^2)^2}{18a^2b^3c^4}$ **25.** $\frac{-16a^3b^2x^4y}{-48a^4bxy^3}$

26. $\frac{(-a)^5b^8}{a^5b^2}$ **27.** $\frac{(4a^{-1})^{-2}}{(2a^4)^2}$ **28.** $\left(\frac{5xy^{-2}}{35x^{-2}y^{-6}}\right)^0$

8-3 Scientific Notation

See pages 425–430.

Concept Summary

- A number is expressed in scientific notation when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

$$a \times 10^n, \text{ where } 1 \le a < 10 \text{ and } n \text{ is an integer.}$$

Examples

1 **Express 5.2×10^7 in standard notation.**

$5.2 \times 10^7 = 52{,}000{,}000$ $n = 7$; move decimal point 7 places to the right.

2 **Express 0.0021 in scientific notation.**

$0.0021 \rightarrow 0002.1 \times 10^n$ Move decimal point 3 places to the right.

$0.0021 = 2.1 \times 10^{-3}$ $a = 2.1$ and $n = -3$

3 **Evaluate $(2 \times 10^2)(5.2 \times 10^6)$. Express the result in scientific and standard notation.**

$(2 \times 10^2)(5.2 \times 10^6) = (2 \times 5.2)(10^2 \times 10^6)$	Associative Property
$= 10.4 \times 10^8$	Product of Powers
$= (1.04 \times 10^1) \times 10^8$	$10.4 = 1.04 \times 10^1$
$= 1.04 \times (10^1 \times 10^8)$	Associative Property
$= 1.04 \times 10^9$ or 1,040,000,000	Product of Powers

Exercises **Express each number in standard notation.** *See Example 1 on page 426.*

29. 2.4×10^5 **30.** 3.14×10^{-4} **31.** 4.88×10^9

Express each number in scientific notation. *See Example 2 on page 426.*

32. 0.00000187 **33.** 796×10^3 **34.** 0.0343×10^{-2}

Evaluate. Express each result in scientific and standard notation.
See Examples 3 and 4 on page 427.

35. $(2 \times 10^5)(3 \times 10^6)$ **36.** $\dfrac{8.4 \times 10^{-6}}{1.4 \times 10^{-9}}$ **37.** $(3 \times 10^2)(5.6 \times 10^{-8})$

8-4 Polynomials

See pages 432–436.

Concept Summary

- A polynomial is a monomial or a sum of monomials.
- A binomial is the sum of *two* monomials, and a trinomial is the sum of *three* monomials.
- The degree of a monomial is the sum of the exponents of all its variables.
- The degree of the polynomial is the greatest degree of any term. To find the degree of a polynomial, you must find the degree of each term.

Examples

1 **Find the degree of $2xy^3 + x^2y$.**

Polynomial	Terms	Degree of Each Term	Degree of Polynomial
$2xy^3 + x^2y$	$2xy^3, x^2y$	4, 3	4

2 **Arrange the terms of $4x^2 + 9x^3 - 2 - x$ so that the powers of x are in descending order.**

$4x^2 + 9x^3 - 2 - x = 4x^2 + 9x^3 - 2x^0 - x^1$	$x^0 = 1$ and $x = x^1$
$= 9x^3 + 4x^2 - x - 2$	$3 > 2 > 1 > 0$

Exercises **Find the degree of each polynomial.** *See Example 3 on page 433.*

38. $n - 2p^2$ **39.** $29n^2 + 17n^2t^2$ **40.** $4xy + 9x^3z^2 + 17rs^3$
41. $-6x^5y - 2y^4 + 4 - 8y^2$ **42.** $3ab^3 - 5a^2b^2 + 4ab$ **43.** $19m^3n^4 + 21m^5n$

Arrange the terms of each polynomial so that the powers of x are in descending order. *See Example 5 on page 433.*

44. $3x^4 - x + x^2 - 5$ **45.** $-2x^2y^3 - 27 - 4x^4 + xy + 5x^3y^2$

8-5 Adding and Subtracting Polynomials

See pages 439–443.

Concept Summary

- To add polynomials, group like terms horizontally or write them in column form, aligning like terms vertically.
- Subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse.

Example **Find $(7r^2 + 9r) - (12r^2 - 4)$.**

$(7r^2 + 9r) - (12r^2 - 4) = 7r^2 + 9r + (-12r^2 + 4)$ The additive inverse of $12r^2 - 4$ is $-12r^2 + 4$.

$= (7r^2 - 12r^2) + 9r + 4$ Group like terms.

$= -5r^2 + 9r + 4$ Add like terms.

Exercises **Find each sum or difference.** *See Examples 1 and 2 on pages 439 and 440.*

46. $(2x^2 - 5x + 7) - (3x^3 + x^2 + 2)$

47. $(x^2 - 6xy + 7y^2) + (3x^2 + xy - y^2)$

48. $(7z^2 + 4) - (3z^2 + 2z - 6)$

49. $(13m^4 - 7m - 10) + (8m^4 - 3m + 9)$

50. $(11m^2n^2 + 4mn - 6) + (5m^2n^2 + 6mn + 17)$

51. $(-5p^2 + 3p + 49) - (2p^2 + 5p + 24)$

8-6 Multiplying a Polynomial by a Monomial

See pages 444–449.

Concept Summary

- The Distributive Property can be used to multiply a polynomial by a monomial.

Examples

1 **Simplify $x^2(x + 2) + 3(x^3 + 4x^2)$.**

$x^2(x + 2) + 3(x^3 + 4x^2) = x^2(x) + x^2(2) + 3(x^3) + 3(4x^2)$ Distributive Property

$= x^3 + 2x^2 + 3x^3 + 12x^2$ Multiply.

$= 4x^3 + 14x^2$ Combine like terms.

2 **Solve $x(x - 10) + x(x + 2) + 3 = 2x(x + 1) - 7$.**

$x(x - 10) + x(x + 2) + 3 = 2x(x + 1) - 7$ Original equation

$x^2 - 10x + x^2 + 2x + 3 = 2x^2 + 2x - 7$ Distributive Property

$2x^2 - 8x + 3 = 2x^2 + 2x - 7$ Combine like terms.

$-8x + 3 = 2x - 7$ Subtract $2x^2$ from each side.

$-10x + 3 = -7$ Subtract $2x$ from each side.

$-10x = -10$ Subtract 3 from each side.

$x = 1$ Divide each side by -10.

Exercises **Simplify.** *See Example 2 on page 444.*

52. $b(4b - 1) + 10b$

53. $x(3x - 5) + 7(x^2 - 2x + 9)$

54. $8y(11y^2 - 2y + 13) - 9(3y^3 - 7y + 2)$

55. $2x(x - y^2 + 5) - 5y^2(3x - 2)$

Solve each equation. *See Example 4 on page 445.*

56. $m(2m - 5) + m = 2m(m - 6) + 16$

57. $2(3w + w^2) - 6 = 2w(w - 4) + 10$

Chapter 8 For More ...
• Extra Practice, see pages 837–839.
• Mixed Problem Solving, see page 860.

8-7 Multiplying Polynomials

See pages 452–457.

Concept Summary

- The FOIL method is the sum of the products of the first terms F, the outer terms O, the inner terms I, and the last terms L.
- The Distributive Property can be used to multiply any two polynomials.

Examples

1 **Find $(3x + 2)(x - 2)$.**

$(3x + 2)(x - 2) = \overset{F}{(3x)(x)} + \overset{O}{(3x)(-2)} + \overset{I}{(2)(x)} + \overset{L}{(2)(-2)}$		FOIL Method
$= 3x^2 - 6x + 2x - 4$		Multiply.
$= 3x^2 - 4x - 4$		Combine like terms.

2 **Find $(2y - 5)(4y^2 + 3y - 7)$.**

$(2y - 5)(4y^2 + 3y - 7)$

$= 2y(4y^2 + 3y - 7) - 5(4y^2 + 3y - 7)$	Distributive Property
$= 8y^3 + 6y^2 - 14y - 20y^2 - 15y + 35$	Distributive Property
$= 8y^3 - 14y^2 - 29y + 35$	Combine like terms.

Exercises **Find each product.** *See Examples 1, 2, and 4 on pages 452–454.*

58. $(r - 3)(r + 7)$ **59.** $(4a - 3)(a + 4)$ **60.** $(3x + 0.25)(6x - 0.5)$
61. $(5r - 7s)(4r + 3s)$ **62.** $(2k + 1)(k^2 + 7k - 9)$ **63.** $(4p - 3)(3p^2 - p + 2)$

8-8 Special Products

See pages 458–463.

Concept Summary

- Square of a Sum: $(a + b)^2 = a^2 + 2ab + b^2$
- Square of a Difference: $(a - b)^2 = a^2 - 2ab + b^2$
- Product of a Sum and a Difference: $(a + b)(a - b) = (a - b)(a + b) = a^2 - b^2$

Examples

1 **Find $(r - 5)^2$.**

$(a - b)^2 = a^2 - 2ab + b^2$	Square of a Difference
$(r - 5)^2 = r^2 - 2(r)(5) + 5^2$	$a = r$ and $b = 5$
$= r^2 - 10r + 25$	Simplify.

2 **Find $(2c + 9)(2c - 9)$.**

$(a + b)(a - b) = a^2 - b^2$	Product of a Sum and a Difference
$(2c + 9)(2c - 9) = 2c^2 - 9^2$	$a = 2c$ and $b = 9$
$= 4c^2 - 81$	Simplify.

Exercises **Find each product.** *See Examples 1, 2, and 4 on pages 459 and 461.*

64. $(x - 6)(x + 6)$ **65.** $(4x + 7)^2$ **66.** $(8x - 5)^2$
67. $(5x - 3y)(5x + 3y)$ **68.** $(6a - 5b)^2$ **69.** $(3m + 4n)^2$

Chapter 8

Practice Test

Vocabulary and Concepts

1. **Explain** why $(4^2)(4^3) \neq 16^5$.
2. **Write** $\frac{1}{5}$ using a negative exponent.
3. **Define and give an example** of a monomial.

Skills and Applications

Simplify. Assume that no denominator is equal to zero.

4. $(a^2b^4)(a^3b^5)$
5. $(-12abc)(4a^2b^4)$
6. $\left(\frac{3}{5}m\right)^2$
7. $(-3a)^4(a^5b)^2$
8. $(-5a^2)(-6b^3)^2$
9. $\frac{mn^4}{m^3n^2}$
10. $\frac{9a^2bc^2}{63a^4bc}$
11. $\frac{48a^2bc^5}{(3ab^3c^2)^2}$

Express each number in scientific notation.

12. 46,300
13. 0.003892
14. 284×10^3
15. 52.8×10^{-9}

Evaluate. Express each result in scientific notation and standard notation.

16. $(3 \times 10^3)(2 \times 10^4)$
17. $\frac{14.72 \times 10^{-4}}{3.2 \times 10^{-3}}$
18. $(15 \times 10^{-7})(3.1 \times 10^4)$

19. **SPACE EXPLORATION** A space probe that is 2.85×10^9 miles away from Earth sends radio signals to NASA. If the radio signals travel at the speed of light (1.86×10^5 miles per second), how long will it take the signals to reach NASA?

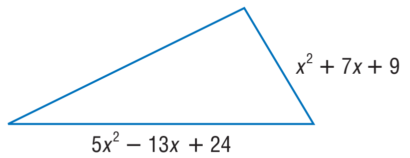
Find the degree of each polynomial. Then arrange the terms so that the powers of y are in descending order.

20. $2y^2 + 8y^4 + 9y$
21. $5xy - 7 + 2y^4 - x^2y^3$

Find each sum or difference.

22. $(5a + 3a^2 - 7a^3) + (2a - 8a^2 + 4)$
23. $(x^3 - 3x^2y + 4xy^2 + y^3) - (7x^3 + x^2y - 9xy^2 + y^3)$

24. **GEOMETRY** The measures of two sides of a triangle are given. If the perimeter is represented by $11x^2 - 29x + 10$, find the measure of the third side.



Simplify.

25. $(h - 5)^2$
26. $(4x - y)(4x + y)$
27. $3x^2y^3(2x - xy^2)$
28. $(2a^2b + b^2)^2$
29. $(4m + 3n)(2m - 5n)$
30. $(2c + 5)(3c^2 - 4c + 2)$

Solve each equation.

31. $2x(x - 3) = 2(x^2 - 7) + 2$
32. $3a(a^2 + 5) - 11 = a(3a^2 + 4)$

33. **STANDARDIZED TEST PRACTICE** If $x^2 + 2xy + y^2 = 8$, find $3(x + y)^2$.
   - (A) 2
   - (B) 4
   - (C) 24
   - (D) cannot be determined

Chapter 8 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. A basketball team scored the following points during the first five games of the season: 70, 65, 75, 70, 80. During the sixth game, they scored only 30 points. Which of these measures changed the most as a result of the sixth game? (Lessons 2-2 and 2-5)

Ⓐ mean

Ⓑ median

Ⓒ mode

Ⓓ They all changed the same amount.

2. A machine produces metal bottle caps. The number of caps it produces is proportional to the number of minutes the machine operates. The machine produces 2100 caps in 60 minutes. How many minutes would it take the machine to produce 5600 caps? (Lesson 2-6)

Ⓐ 35 Ⓑ 58.3 Ⓒ 93.3 Ⓓ 160

3. The odometer on Juliana's car read 20,542 miles when she started a trip. After 4 hours of driving, the odometer read 20,750 miles. Which equation can be used to find r, her average rate of speed for the 4 hours? (Lesson 3-1)

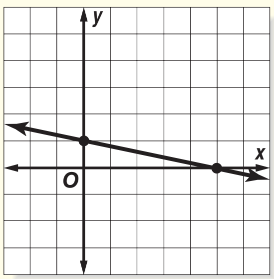
Ⓐ $r = 20{,}750 - 20{,}542$

Ⓑ $r = 4(20{,}750 - 20{,}542)$

Ⓒ $r = \frac{20{,}750}{4}$

Ⓓ $r = \frac{20{,}750 - 20{,}542}{4}$

4. Which equation best describes the graph? (Lesson 5-4)



Ⓐ $y = -\frac{1}{5}x + 1$

Ⓑ $y = -5x + 1$

Ⓒ $y = \frac{1}{5}x + 5$

Ⓓ $y = -5x - 5$

5. Which equation represents the line that passes through the point at $(-1, 4)$ and has a slope of -2? (Lesson 5-5)

Ⓐ $y = -2x - 2$ Ⓑ $y = -2x + 2$

Ⓒ $y = -2x + 6$ Ⓓ $y = -2x + 7$

6. Mr. Puram is planning an addition to the school library. The budget is \$7500. Each bookcase costs \$125, and each set of table and chairs costs \$550. If he buys 4 sets of tables and chairs, which inequality shows the number of bookcases b he can buy? (Lesson 6-6)

Ⓐ $4(550) + 125b \le 7500$

Ⓑ $125b \le 7500$

Ⓒ $4(550 + 125)b \le 7500$

Ⓓ $4(125) + 550b \le 7500$

7. Sophia and Allie went shopping and spent \$122 altogether. Sophia spent \$25 less than twice as much as Allie. How much did Allie spend? (Lesson 7-2)

Ⓐ \$39 Ⓑ \$49 Ⓒ \$53 Ⓓ \$73

8. The product of $2x^3$ and $4x^4$ is (Lesson 8-1)

Ⓐ $8x^{12}$. Ⓑ $6x^{12}$. Ⓒ $6x^7$. Ⓓ $8x^7$.

9. If 0.00037 is expressed as 3.7×10^n, what is the value of n? (Lesson 8-3)

Ⓐ -5 Ⓑ -4 Ⓒ 4 Ⓓ 5

10. When $x^2 - 2x + 1$ is subtracted from $3x^2 - 4x + 5$, the result will be (Lesson 8-5)

Ⓐ $2x^2 - 2x + 4$. Ⓑ $2x^2 - 6x + 4$.

Ⓒ $3x^2 - 6x + 6$. Ⓓ $4x^2 - 6x + 6$.

The Princeton Review **Test-Taking Tip**

Question 5 When you write an equation, check that the given values make a true statement. For example, in Question 5, substitute the values of the coordinates $(-1, 4)$ into your equation to check.

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Find the 15th term in the arithmetic sequence $-20, -11, -2, 7, \ldots$. (Lesson 4-7)

12. Write a function that includes all of the ordered pairs in the table. (Lesson 4-8)

x	−3	−1	1	3	4
y	12	4	−4	−12	−16

13. Find the y-intercept of the line represented by $3x - 2y + 8 = 0$. (Lesson 5-4)

14. Graph the solution of the linear inequality $3x - y \leq 2$. (Lesson 6-6)

15. Let $P = 3x^2 - 2x - 1$ and $Q = -x^2 + 2x - 2$. Find $P + Q$. (Lesson 8-5)

16. Find $(x^2 + 1)(x - 3)$. (Lesson 8-7)

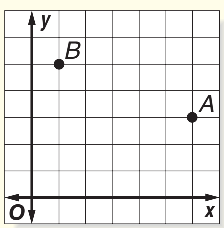
Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- **(A) the quantity in Column A is greater,**
- **(B) the quantity in Column B is greater,**
- **(C) the two quantities are equal, or**
- **(D) the relationship cannot be determined from the information given.**

Column A	Column B

17.



the domain of point *A*	the range of point *B*

(Lesson 4-3)

18.

$4x - 10 \geq 20$	$\frac{-6(x-1)}{8} \geq 3$

(Lesson 6-3)

19.

the x value in the solution of $x - 3y = 2$ and $x + 3y = 0$	the x value in the solution of $3x + 8y = 6$ and $x - 8y = 2$

(Lesson 7-3)

20.

$\frac{2b^{-3}c^2}{4bc}$	$\frac{10b^4}{20b^8c^{-1}}$

(Lesson 8-2)

21.

5.01×10^{-2}	50.1×10^{-4}

(Lesson 8-3)

22.

the degree of $x^2 + 5 - 6x + 13x^3$	the degree of $10 - y - 2y^2 - 4y^3$

(Lesson 8-4)

23. $m^2 + n^2 = 10$ and $mn = -6$

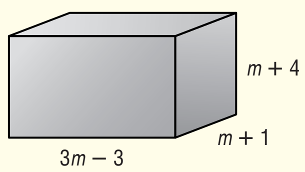
$(m + n)^2$	$(m - n)^2$

(Lesson 8-8)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

24. Use the rectangular prism below to solve the following problems. (Lessons 8-1 and 8-7)



a. Write a polynomial expression that represents the surface area of the top of the prism.

b. Write a polynomial expression that represents the surface area of the front of the prism.

c. Write a polynomial expression that represents the volume of the prism.

d. If $m = 2$ centimeters, then what is the volume of the prism?

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